Passive Rational Interpolation-Based Reduction via Carathéodory Extension for General Systems

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***Abstract*—Passive reduction of interconnects requires the system to be described in the so-called passive form, which limits the application of existing approaches. Passive reduction of a dynamic system in general structure form still remains a difficult problem, and existing solutions are expensive. This brief presents a novel rational interpolation-based reduction framework for reducing the dynamic systems described in any internal structure. The new method is based on the Carathéodory extension, which ensures that the interpolating function is passive without any restriction on the circuit structure. As an explicit moment-matching method, it has a similar computational cost as the Krylov-subspace-based reduction methods and is very efficient to reduce the large systems whose input–output behavior can be approximated by a small number of moments.**

***Index Terms*—Model order reduction (MOR), passivity.**

I. Introduction

MODEL ORDER reduction (MOR) by Krylov subspace methods has been proven to be an efficient technique for reducing the complexity of interconnects [1], [2]. For passive reduction of interconnect circuits, existing approaches, however, require that the system must be formulated in passive form, i.e., a state-space representation with positive semidefinite system matrices and the same input and output mapping matrices. The passivity is ensured by the congruency transformation, which can preserve the passive form in the reduced model. Those restrictions are generally satisfied for interconnects modeled as RLC circuits. However, many passive systems are not conveniently put into such a form [3].

In fact, passivity is more generally characterized by the positive realness of the transfer function [4]. In order to preserve passivity for systems with arbitrary internal structure, the positive realness of the transfer function of the reduced system should be enforced. Toward this goal, usually, positive-real truncated balanced realization algorithms [3] have to be performed. However, their high computational cost $O(n^3)$ (n is the order of the system to be reduced) makes them infeasible for large-scale systems in practice.

Over the last several years, a new theory of positive real analytic interpolation with complexity constraint has been developed for both scalar and matrix interpolating functions (or interpolant for short) in discrete-time setting [5]–[7]. The problem is interpolating prescribed values and successive derivatives on a given set of points in the unit disc by means of a strictly *positive real* rational function in the unit disc. One special case is the *Carathéodory extension*, which specifies the interpolation conditions at the origin up to a number of derivatives.

In this brief, we propose a novel Carathéodory-extension-based model reduction scheme. The new method, which is called *CEMOR*, can generate guaranteed passive reduced models of dynamic systems with arbitrary internal structure and formulations. The reduced model will agree with the original model up to a number of moments at an expansion point. In the proposed method, we first choose an expansion point and compute the moments of the original system at that point as the interpolation conditions with derivatives. Then, we transform the interpolation conditions to the discrete-time domain, obtain the central solution of Carathéodory extension, and transform the interpolant back to the continuous-time domain as the reduced system. The proposed rational interpolation method has a similar computational cost as the Krylov subspace methods but can generate guaranteed passive reduced models for systems with arbitrary internal structure.

This brief is organized as follows. In Section II, we review the background of MOR. In Section III, we introduce the Carathéodory extension problem in discrete-time domain. The new method CEMOR is described in Section IV and generalized to the matrix-valued case in Section V. Several experimental results are reported in Section VI.

Manuscript received November 15, 2009; revised February 24, 2010; accepted April 21, 2010. Date of current version September 15, 2010. This work was supported in part by the National Science Foundation (NSF) Award under Grant CCF-0448534, by the NSF under Grant OISE-0929699, and by the National Natural Science Foundation of China under Grant 60828008. This paper was recommended by Associate Editor P. Li.

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Digital Object Identifier 10.1109/TCSII.2010.2056010

II. Background

A. MOR in a Nutshell

Given a state-space model in descriptor form

$$E\dot{x}(t) = Ax(t) + Bu(t) \quad y(t) = Cx(t) + Du(t) \tag{1}$$

where $E, A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times p}$, $C \in \mathbb{R}^{p\times n}$, $D \in \mathbb{R}^{p\times p}$, and $y(t), u(t) \in \mathbb{R}^p$, model reduction algorithms seek to produce a similar system

$$E_r\dot{\tilde{x}}(t) = A_r\tilde{x}(t) + B_r u(t) \quad \tilde{y}(t) = C_r\tilde{x}(t) + D_r u(t) \tag{2}$$

where $E_r, A_r \in \mathbb{R}^{r\times r}$, $B_r \in \mathbb{R}^{r\times p}$, and $C_r \in \mathbb{R}^{p\times r}$, of order r that is much smaller than the original order n, but for which the outputs $y(t)$ and $\tilde{y}(t)$ are approximately equal for inputs $u(t)$ of interest. Often, the transfer functions $H(s) = D + C(sE - A)^{-1}B$ and $H_r(s) = D_r + C_r(sE_r - A_r)^{-1}B_r$ are used as a metric for approximation.

B. Passivity and Positive Realness

Passivity is an important property of many physical systems. The system is passive if and only if its transfer function $H(s)$ is positive real [4], which yields three observations, where $\overline{H}$ denotes complex conjugate, H^H denotes Hermitian (complex conjugate and transpose), and ≥ 0 denotes positive semidefiniteness in a matrix context.

1) $H(s)$ is analytic for $Re(s) > 0$.
2) $\overline{H(s)} = H(\bar{s})$ for $s \in \mathbb{C}$.
3) $H(s) + H(s)^H \geq 0$ for $Re(s) > 0$.

C. Krylov Subspace and Interpolation

Recently developed model reduction methods suitable for application to large systems are based on Krylov subspace techniques. The reduced model is obtained via a projection $E_r = V^TEV,\ A_r = V^TAV,\ B_r = V^TB,\ C_r = CV$, where the columns of V span a Krylov subspace. Because of the moment-matching properties of Krylov subspace, $H_r(s)$ will agree with $H(s)$ up to the first m derivatives on s_0

$$H_r^{(k)}(s_0) = H^{(k)}(s_0), \quad (k = 0, 1, \ldots, m). \tag{3}$$

Hence, the Krylov subspace method is indeed an approach to interpolation with derivatives. However, unless a special internal structure is available, there is no guarantee that the interpolant $H_r(s)$ is positive real in general.

III. CARATHÉODORY EXTENSION

A. Problem Statement

In this section, we present the classical Carathéodory extension problem, which is derived in the discrete-time domain.

The Carathéodory extension problem is a special case of the general Nevanlinna–Pick interpolation problem with derivative constraints, where multiple expansion points are allowed [6]. If 0 is chosen as the only expansion point, the general problem is reduced to Carathéodory extension, which is considered in this brief.

Given a scalar sequence $(w_0, w_1, \ldots, w_m)$, the Carathéodory extension problem with degree constraint amounts to determining any function $f(z)$ satisfying three conditions.

1) $f(z)$ fulfills the interpolation constraints

$$\frac{f^{(k)}(0)}{k!} = w_k, \quad (k = 0, 1, \ldots, m). \tag{4}$$

2) $f(z)$ is strictly positive real, i.e., f is analytic in the closed unit disc $\bar{\mathbb{D}}$, where $(\mathbb{D} = \{z : |z| < 1\})$, and $Ref(z) > 0$ for all $z \in \bar{\mathbb{D}}$.
3) f is rational, and the degree $(\deg f(z)) \leq m$.

There exists an interpolant for the interpolation problem with derivative constraints if and only if a symmetric Toeplitz matrix

$$P = \begin{bmatrix} 2w_0 & w_1 & \cdots & w_m \\ w_1 & 2w_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & w_1 \\ w_m & \cdots & w_1 & 2w_0 \end{bmatrix} \tag{5}$$

is nonnegative definite (a special case of Theorem 2.1 [6]).

Note that, if $(w_0, w_1, \ldots, w_m)$ are the moments of a high-order system in discrete-time domain, $f(z)$ can be the transfer function of a reduced system of order m, and the reduced system must be passive as $f(z)$ is positive real.

B. Determination of Interpolating Function

The problem is determining a rational interpolant of degree m

$$f(z) = \frac{\beta(z)}{\alpha(z)} = \frac{\beta_0 + \beta_1 z + \cdots + \beta_m z^m}{\alpha_0 + \alpha_1 z + \cdots + \alpha_m z^m} \tag{6}$$

which satisfies the three conditions.

Given $f(z)$, the *spectral density* of $f(z)$ is given by

$$\begin{aligned}\Phi(z) = f(z) + f(z^{-1}) &= \frac{\alpha(z)\beta(z^{-1}) + \alpha(z^{-1})\beta(z)}{\alpha(z)\alpha(z^{-1})} \\ &= \frac{\rho(z)\rho(z^{-1})}{\alpha(z)\alpha(z^{-1})}.\end{aligned} \tag{7}$$

There exists a bijective (one-to-one and onto) map between the set of pairs of real polynomials

$$\{(\alpha(z), \beta(z)) : \deg(\alpha(z)) \leq m, \deg(\beta(z)) \leq m\}, \quad \alpha(0) \neq 0 \quad \beta(0) \neq 0 \tag{8}$$

and the set of real stable polynomials

$$\left\{\rho(z) : \deg(\rho(z)) = m, \rho(z) \neq 0, \forall z \in \bar{\mathbb{D}}\right\} \tag{9}$$

where $\rho(z) = \rho_0 + \rho_1 z + \cdots + \rho_m z^m$. A stable polynomial here means that all the roots are outside the unit circle.

The bijectivity implies that the roots of $\rho(z)$, i.e., *spectral zeros*, which are the zeros of spectral density, are the characterizing factor. In other words, if a set of spectral zeros is assigned, $f(z)$ can be uniquely determined by satisfying the three conditions.

Specifically, the computation of an interpolant $f(z)$ from $\rho(z)$ amounts to an optimization problem $\min_{\alpha \in S_m} J_\rho(\alpha)$

$$J_\rho(\alpha) = \boldsymbol{\alpha}^T P \boldsymbol{\alpha} - 2\left\langle \log(\alpha(z)), \rho(z)\rho(z^{-1})\right\rangle \tag{10}$$

where $\boldsymbol{\alpha}^T = [\alpha_0 \ \cdots \ \alpha_m] \in \mathbb{R}^{1\times(m+1)}$, and $\langle f(z), g(z)\rangle = (1/2\pi)\int_{-\pi}^{\pi} f^*(e^{i\theta})g(e^{i\theta})\,d\theta$ defines the inner product of two functions $f(z)$ and $g(z)$. If the coefficients are real, $f^*(z) = f(z^{-1})$, which is the case for our problem.

The derivation of such optimization problem for the general Nevanlinna–Pick interpolation problem with derivative constraints is given in [6]. It is easy to see that, if 0 is chosen as the only expansion point, the optimization problem in [6] will reduce to the preceding optimization problem (10).

Given a set of spectral zeros, the interpolant can be determined by an optimization problem (10). As any interpolant satisfying the interpolation conditions meets our needs for passive order reduction, a special case, i.e., the *central or maximum entropy solution*, is of particular interest to us due to the simple computational procedure. The maximum entropy solution can be determined by solving a linear system of equations, instead of the optimization problem.

C. Maximum Entropy Solution

Now, let us consider the maximum entropy solution, i.e., the special case of the problem in which all the spectral zeros are assigned at infinity. In this special case, $\rho(z)\rho(z^{-1}) = 1$, and hence, $\langle \log(\alpha(z)), \rho(z)\rho(z^{-1})\rangle = \log(\alpha_0)$. Consequently, the objective function becomes

$$J_1(\alpha) = \boldsymbol{\alpha}^T P \boldsymbol{\alpha} - 2\log(\alpha_0). \tag{11}$$

If the Toeplitz matrix P is positive definite, J_1 is strictly convex. Hence, there is at most one minimum. To determine this possible minimum, set the gradient equal to zero to obtain

$$\nabla J_1 = \begin{bmatrix} 2w_0 & w_1 & \cdots & w_m \\ w_1 & 2w_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & w_1 \\ w_m & \cdots & w_1 & 2w_0 \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_m \end{bmatrix} - \begin{bmatrix} 1/\alpha_0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = 0 \tag{12}$$

which can be written as $P\boldsymbol{\alpha} = \boldsymbol{e}(1/\alpha_0)$, where $\boldsymbol{e} = [1, 0, \ldots, 0]^T$. Note that, since $\alpha_0 = \boldsymbol{e}^T\boldsymbol{\alpha} = \boldsymbol{e}^T P^{-1}\boldsymbol{e}(1/\alpha_0)$, we have

$$\alpha_0 = \sqrt{\boldsymbol{e}^T P^{-1} \boldsymbol{e}}. \tag{13}$$

With α_0, $\boldsymbol{\alpha}$ is the unique solution of a linear system of equations (12), because P is positive definite.

Finally, given $\alpha(z)$, $\beta(z)$ can be solved by

$$\alpha(z)\beta(z^{-1}) + \alpha(z^{-1})\beta(z) = \rho(z)\rho(z^{-1}) \tag{14}$$

where $\rho(z)\rho(z^{-1}) = 1$ due to the maximum entropy solution. Identifying coefficients of the same power in z, we can come up with the following linear equations:

$$\left(\begin{bmatrix} \alpha_0 & \alpha_1 & \cdots & \alpha_m \\ \alpha_1 & \cdots & \alpha_m & 0 \\ \vdots & \ddots & \ddots & \vdots \\ \alpha_m & 0 & \cdots & 0 \end{bmatrix} + \begin{bmatrix} \alpha_0 & \alpha_1 & \cdots & \alpha_m \\ 0 & \alpha_0 & \cdots & \alpha_{m-1} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \alpha_0 \end{bmatrix} \right) \times \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_m \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{15}$$

which can also be written as $(H_\alpha + T_\alpha)\boldsymbol{\beta} = \boldsymbol{e}$.

IV. New Reduction Method: CEMOR

A. Algorithm Flow

We present the new model reduction method based on single-input–single-output system $h(s)$. However, the results can be generalized to the multi-input–multi-output case, as shown in the next section. We first give an overall flow of the algorithm CEMOR and then present the important steps in detail.

1) Choose an expansion point $\sigma \in R^+$.
2) Perform frequency scaling to normalize the expansion point to 1 by (18).
3) Generate moments at normalized expansion point 1 by (20).
4) Transform moments to the discrete-time domain by (24).
5) Compute the Toeplitz matrix using (5).
6) Check P for positive definite property.
7) Solve (12) to obtain $\alpha(z)$.
8) Solve (15) to obtain $\beta(z)$.
9) Realize $f(z)$ by any canonical form.
10) Transform interpolant back to the continuous-time domain by (27).
11) Perform reverse frequency scaling by (28).

In CEMOR, steps 1–3 compute the moments from the original circuits and perform the scaling as required by the new method. Step 4 and Step 10 transform the information between continuous- and discrete-time domains. Steps 6–8 compute the reduced model $f(z)$ in the discrete-time domain.

B. Frequency Scaling

We choose a positive real expansion point $\sigma \in R^+$. Typically, the point chosen in higher dynamic frequency range will result in a more compact model. Now, we normalize the expansion point to 1 by frequency scaling

$$h(s) = D + C(sE - A)^{-1}B = D + C\left(\frac{s}{\sigma}(\sigma E) - A\right)^{-1} B \tag{16}$$

which results in the following system with expansion point $\tilde{\sigma} = 1$:

$$h(\tilde{s}) = D + C(\tilde{s}\tilde{E} - A)^{-1}B \tag{17}$$

where $\tilde{s} = s/\sigma$ and

$$\tilde{E} = \sigma E. \tag{18}$$

Scaling the expansion point to 1 will ensure a good numerical condition in the bilinear transformation process.

C. Moment Generation and Passive Condition

Given the dynamic system (17) and the expansion point $\tilde{\sigma} = 1$, defining

$$\mathcal{A} = (A - \tilde{\sigma}\tilde{E})^{-1}\tilde{E} \qquad \mathcal{R} = -(A - \tilde{\sigma}\tilde{E})^{-1}B \tag{19}$$

the moments at the expansion point have the following formula:

$$m_0 = C\mathcal{R} + D \quad m_i = C\mathcal{A}^i\mathcal{R}, \quad (i \geq 1). \tag{20}$$

For the reduced model $h_r(s)$, we have three requirements.

1) $h_r(s)$ fulfills the interpolation constraints

$$\frac{h_r^{(k)}(\tilde{\sigma})}{k!} = m_k, \quad (k = 0, 1, \ldots, r). \tag{21}$$

2) $h_r(s)$ is strictly positive real, i.e., $h(s)$ is analytic in the right-hand plane $\{s : Re(s) \geq 0\}$ and $Re(h_r(s)) > 0$ for all $\{s : Re(s) \geq 0\}$.
3) $h_r(s)$ is rational, and $\deg(h_r(s)) \leq r$.

If $h_r(s)$ satisfies the three conditions, it is the transfer function of the desired reduced model, which is passive and accurate to the rth moment of the original transfer function.

D. Transformation to Discrete-Time Domain

The Carathéodory extension problem considered in the previous section is assumed to find the mapping $f(z)$ from the unit disc onto the right half-plane. In the continuous-time domain, we need to find a transfer function $h_r(s)$ from the right half-plane to the right half-plane. In this case, we need to transform the interpolation data (interpolation point and moments) from the right-hand plane to the unit disc first.

Thus, we need a bilinear transformation, which maps the right-half plane $\{s : Re(s) > 0\}$ to the unit disc $\{z : |z| < 1\}$ and maps interpolation point $s = 1$ to $z = 0$. The following bilinear transformation will achieve this [6]:

$$z(s) = \frac{-s+1}{s+1} \quad s(z) = \frac{-z+1}{z+1}. \tag{22}$$

Under the bilinear transformation, we have $f(z) = h_r(s(z))$ and $f(0) = h_r(1)$. Given

$$\frac{f^{(k)}(0)}{k!} = w_k \quad \frac{h_r^{(k)}(1)}{k!} = m_k \tag{23}$$

for $(k = 0, 1, \ldots, r)$, the derivatives $f^{(k)}(0)$ is related to the derivatives $h_r^{(k)}(1)$ as follows [6]:

$$\begin{aligned} w_0 &= m_0 \\ w_k &= \frac{1}{k!}\sum_{l=1}^{k}\binom{k}{l} m_{k-l+1}(k-l+1)!\left(s^{(1)}(0)\right)^{k-l} s^{(l)}(0). \end{aligned} \tag{24}$$

The coefficients $\binom{k}{l}$ are binomial coefficients, which fulfill the recursive formula

$$\begin{aligned} \binom{k}{l} &= 1, \qquad (l = 1, k) \\ \binom{k}{l} &= \frac{2k-l}{l}\binom{k-1}{l-1} + \binom{k-1}{l}, \qquad (1 < l < k). \end{aligned} \tag{25}$$

The term $s^{(l)}(0)$ is obtained by

$$s^{(l)}(0) = 2(-1)^1 l!. \tag{26}$$

Now, we can use the Carathéodory extension method in the previous section to obtain $f(z)$ from $w_0, \ldots, w_r$.

E. Transformation Back to Continuous-Time Domain

Given the transfer function $f(z)$, it can be realized by any canonical state-space form (A_f, B_f, C_f, D_f), which can be transformed back to the continuous-time domain by the following transformation derived from (22):

$$\begin{aligned} A_r &= (I - A_f)(I + A_f)^{-1} \\ B_r &= -2(I + A_f)^{-1}B_f \\ C_r &= C_f(I + A_f)^{-1} \\ D_r &= -C_f(I + A_f)^{-1}B_f + D_f. \end{aligned} \tag{27}$$

Thus, the reduced model is given by $(E_r, A_r, B_r, C_r, D_r)$, where

$$E_r = \frac{1}{\sigma}I_r \tag{28}$$

is the inverse process of the frequency scaling in (18).

F. Complexity Analysis and Limitations

The cost to generate moments is $O(n^\alpha)$, which is similar to Krylov subspace methods (α depends on the sparsity of the system and $1 < \alpha < 2$ for most cases of interest). The rest of the procedures only require $O(r^3)$, where r is the order of the reduced system. Since $r \ll n$, the cost is dominated by $O(n^\alpha)$.

V. Extension to MIMO Systems

Given an interpolation point $z_0 = 0$ and a set of m matrix-valued interpolation values $(W_0, W_1, \ldots, W_m) \subset \mathbb{R}^{p\times p}$, where W_0 is assumed to be symmetric (the nonsymmetric case can be transformed to symmetric, as shown at the end of this section), the matrix-valued Carathéodory extension problem with degree constraint amounts to determining any function $F(z)$ of order $r = mp$ fulfilling the interpolation constraints

$$\frac{1}{k!}F^{(k)}(0) = W_k, \quad (k = 0, 1, \ldots, m). \tag{29}$$

$F(z)$ is strictly positive real, i.e., $F(z)$ is analytic in the closed unit disc $\bar{\mathbb{D}}$ and

$$Re\,(F(z)) = \frac{1}{2}\left(F(z) + F(z^{-1})^T\right) > 0 \tag{30}$$

for all $z \in \bar{\mathbb{D}}$. There exists an interpolant for the interpolation problem with derivative constraints if and only if a symmetric block Toeplitz matrix

$$\mathbf{\Pi} = \begin{bmatrix} W_0 + W_0^T & W_1^T & \cdots & W_m^T \\ W_1 & W_0 + W_0^T & \ddots & \vdots \\ \vdots & \ddots & \ddots & W_1^T \\ W_m & \cdots & W_1 & W_0 + W_0^T \end{bmatrix} \tag{31}$$

is nonnegative definite. In the MIMO case, the cost function (10) is generalized as

$$J_P(\mathbf{R}) = \mathbf{trace}(\mathbf{R^T \Pi R}) - \mathbf{2}\left\langle \log\left(\det\left(\mathbf{R}(\mathbf{z})\right)\right), \rho(\mathbf{z})\rho(\mathbf{z^{-1}})\right\rangle \tag{32}$$

where

$$\mathbf{R} = \begin{bmatrix} R_0 \\ \vdots \\ R_n \end{bmatrix} \in \mathbb{R}^{p(m+1)\times p} \tag{33}$$

and R_0 is assumed to be an upper triangular matrix. $R_0, \ldots, R_n$ are the coefficients of the $p \times p$ matrix polynomial

$$R(z) = R_0 + R_1 z + \cdots + R_n z^n \tag{34}$$

which is the generalization of $\alpha(z)$ in scalar case. Similarly, as for central solution, where $\rho(z)\rho(z^{-1}) = 1$, the objective function becomes

$$J_1(\mathbf{R}) = \mathbf{trace}(\mathbf{R^T \Pi R}) - \mathbf{2}\log\left(\det(\mathbf{R_0})\right). \tag{35}$$

Setting the gradient of J_1 equal to zero, we obtain

$$\mathbf{\Pi R} = \mathbf{E}R_0^{-T} \tag{36}$$

where $\mathbf{E} = [I, 0, \ldots, 0]^T$. Note that, since $R_0 = \mathbf{E}^T\mathbf{R} = \mathbf{E}^T\mathbf{\Pi}^{-1}\mathbf{E}R_0^{-T}$, we have

$$R_0 R_0^T = \mathbf{E}^T \mathbf{\Pi}^{-1}\mathbf{E}. \tag{37}$$

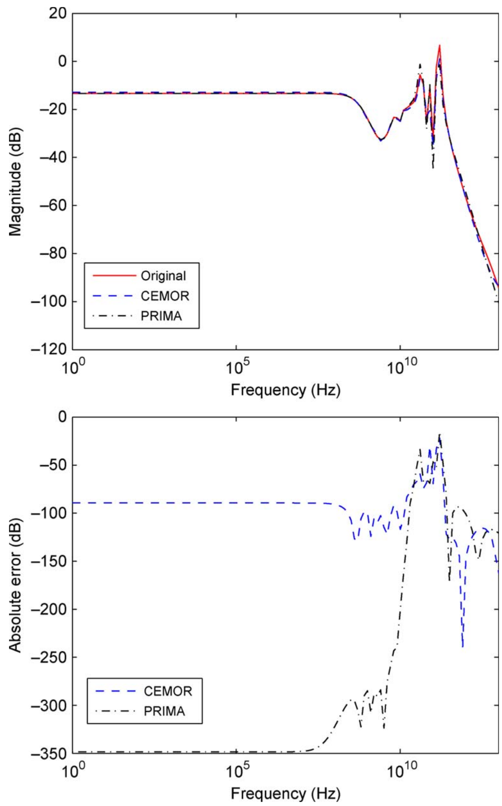

Fig. 1. Magnitude responses and errors of the first example.

By performing Cholesky factorization, we can obtain R_0. With R_0, $\mathbf{R}$ is the unique solution of a linear system of equations (36), because $\mathbf{\Pi}$ is positive definite.

The spectral density of $F(z)$ can be factorized as follows:

$$\Phi(z) = F(z) + F(z^{-1}) = V(z^{-1})^T V(z) \tag{38}$$

which is the generalization of (7) in scalar case. $V(z) = \rho(z)R(z)^{-1}$ is the *spectral factor* of $\Phi(z)$. For maximum-entropy solution, we have $\rho(z) = 1$ and $V(z) = R(z)^{-1}$, which can be realized by canonical form. Given any minimal realization of $V(z)$

$$V(z) = zC_v(I - zA_v)^{-1}B_v + D_v \tag{39}$$

there is a unique $F(z)$ satisfying (38) [7]

$$F(z) = 2z\left(B_v^T X A_v + D_v^T C_v\right)(I - zA_v)^{-1}B_v + B_v^T X B_v + D_v^T D_v \tag{40}$$

where X is the unique solution to the Lyapunov equation

$$A_v^T X A_v - X + C_v^T C_v = 0. \tag{41}$$

Note that, instead of $O(n^3)$, the cost of the Lyapunov equation here is $O((mp)^3)$, which is not expensive because $pm \ll n$.

For nonsymmetric W_0, perform SVD on W_0 as $W_0 = USZ^T$, and transform W_i to $\tilde{W}_i$ by $\tilde{W}_i = U^T W_i Z$ such that $\tilde{W}_0$ is symmetric. After obtaining the interpolant $\tilde{F}(z)$ from $\tilde{W}_i$, the interpolant $F(z)$ from W_i can be obtained as $F(z) = U\tilde{F}(z)Z^T$.

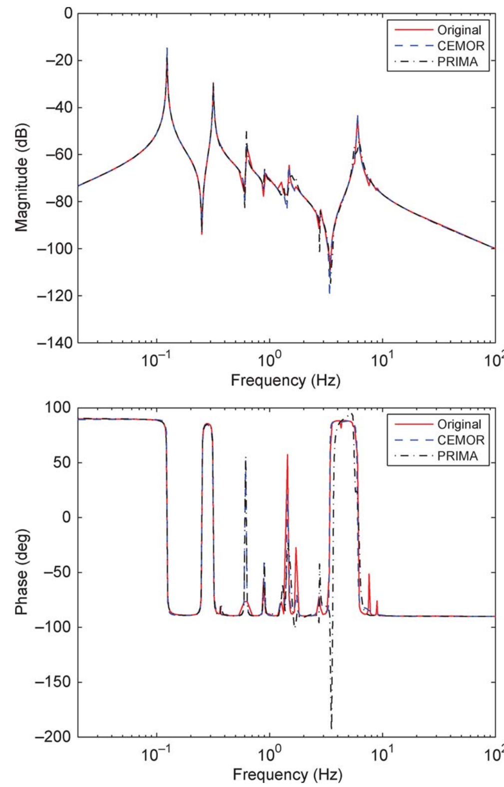

Fig. 2. Magnitude and phase responses of the second example.

VI. Experimental Results

The proposed method has been implemented in Matlab 7.0 and tested on an Intel quad-core workstation with 16-GB memory.

The first example is an RLC mesh of order 1720. We compare CEMOR with PRIMA in Fig. 1. Given an expansion point of 100 GHz and reduced order of 30, although both methods work well, PRIMA has a smaller error as it is based on a numerically superior Arnoldi process. However, PRIMA cannot be relied to preserve passivity for general structure systems.

To illustrate this, we use a non-RLC example. The example is a structural model of the International Space Station [8]. This example has an order of 270, and the reduced orders are 60. The magnitude and phase responses of the first port are shown in Fig. 2. From the magnitude responses, both methods work well. Note that the phase response of CEMOR are between $-90°$ and $90°$, which means that the real part of the transfer function is nonnegative and the reduced model is passive. However, the PRIMA reduced model is not passive as the phase response goes above $90°$ and drops below $-90°$ at several frequency ranges, which can be easily observed.

VII. Conclusion

In this brief, a novel positive real rational interpolation-based approach has been proposed to generate guaranteed passive reduced models for general systems. The proposed method has a similar moment-matching property and similar computational cost as the Krylov subspace methods. As an explicit moment-matching method, it is very efficient to reduce the large systems whose input–output behavior can be approximated by a small number of moments.

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