Second-Order Balanced Truncation for Passive Order Reduction of RLCK Circuits

Boyuan Yan, *Student Member, IEEE*, Sheldon X.-D. Tan, *Senior Member, IEEE* Bruce McGaughy, *Member, IEEE*

***Abstract*— In this paper, we propose a novel model order reduction approach, SBPOR (Second-order Balanced truncation for Passive Order Reduction), which is the first second-order balanced truncation method proposed for passive reduction of RLCK circuits. By exploiting the special structure information in the circuit formulation, second-order Gramians are defined based on a symmetric first-order realization in descriptor from. As a result, SBPOR can perform the traditional balancing with passivity-preserving congruency transformation at the cost of solving one generalized Lyapunov equation. Owing to the second-order formulation, SBPOR also preserves the structure information inherent to RLCK circuits. We further propose, SOGA (Second-Order Gramian Approximation version of SB-POR), to mitigate high computational cost of solving Lyapunov equation. Experimental results demonstrate that SBPOR and SOGA are globally more accurate than the Krylov subspace based approaches.**

***Index Terms*— Model order reduction, Simulation, Krylov subspace, Projection, Truncated balanced realization.**

I. Introduction

Model order reduction (MOR) is an efficient technique to reduce the circuit complexity while producing a good approximation of the input-output behavior [7], [10], [14], [15]. When an RLCK circuit is formulated in the second-order form, inductance (or partial inductance) will be represented in its inverse form, which is called susceptance. Susceptance coupling are shown to be more localized than inductance coupling and its matrix is diagonally dominant like capacitance matrix [1]. Hence, susceptance matrix can be sparsified much easily without loss of stability, which, however, is difficult in general for the inductance matrix [2]. The new susceptance element (called ”K” element) can be stamped back into the circuit matrix using the SPICE-compatible equivalent circuits [3]. Model order reduction techniques for second-order systems, which are more suitable for reducing RLCK circuits, have been developed in the past [11], [13].

However, existing second-order MOR techniques are mainly based on Krylov-subspace methods, which in general have difficulties to generate reduced models with global accuracy. Therefore, another approach, truncated balanced realization (TBR), or balanced truncation (BT), which was originally developed in the control community [6], has been studied intensively for interconnect reduction recently [8], [9], [12], [16]–[18]. The idea of TBR method is to first transform an original system into a new coordinate such that each state in this coordinate is equally controllable and observable before the consequent truncation of the weak states. To perform the passive reduction, positive-real TBR (PR-TBR) was applied in [8], which solves more expensive quadratic matrix equations. PR-TBR has no constrains on the internal structure of the state-space equations. But it also does not preserve any structure information inherent to RLCK circuits such as symmetry, positive semi-definiteness and sparsity, during the reduction process. Another issue is that existing balanced truncation techniques for interconnect reduction are first-order based and cannot handle RLCK circuits formulated as second-order systems.

In the control literature [5], Meyer and Srinivasan introduced a second-order balanced truncation method where second-order Gramians are defined based on Moore's first-order balanced truncation method. However, in order to preserve the stability of original system, congruency transformation instead of similarity transformation is performed. As a result, the transformed system is not really balanced, which sacrifices the accuracy.

In this paper, we propose a new balanced truncation method, SBPOR (Second-order Balanced truncation for Passive Order Reduction), for passive reduction of RLCK circuits. By exploiting the symmetric positive definiteness of the system matrices in the second-order circuit formulation, the new approach resolves the issue existing in [5] by defining second-order Gramians based on a symmetric first-order realization. As a result, balancing and reduction can be achieved via only congruency transformation without any accuracy degradation. In contrast to the first-order balanced truncation approaches, SBPOR can also preserve the structure information inherent to RLCK circuits and only needs to solve one linear matrix equation instead of two quadratic matrix equations. Furthermore, to mitigate the high computational cost of solving Lyapunov equation, a Second-Order Gramian Approximation version, SOGA, is proposed to generalize the existing first-order Gramian approximation technique PMTBR [9] to second-order systems.

II. First-order balanced truncation

Consider a linear time-invariant (LTI) stable system in a standard state-space form

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $A \in R^{n\times n}$, $B \in R^{n\times p}$, $C \in R^{q\times n}$, and in which $x(t)$ is the state vector, and $u(t)$ and $y(t)$ represent the input and output, respectively. The controllability and observability Gramians are computed from the Lyapunov equations

$$\begin{aligned} AX + XA^T + BB^T = 0 \\ A^TY + YA + C^TC = 0 \end{aligned} \tag{2}$$

Manuscript received October 2007 and revised February 2008. This paper was recommended by Associate Editor D.Z. Pan.

Boyuan Yan and Sheldon X.-D. Tan are with Department of Electrical Engineering, University of California, Riverside, CA 92521 USA (e-mail: {byan,stan}@ee.ucr.edu)

Bruce McGaughy is with Cadence Design System, Cadence Design Systems Inc., San Jose, California 95134, USA.

This work is funded in part by NSF CAREER Award under grant No. CCF-0448534, in part by UC MICRO awards #06-252 and #07-105 via Cadence Design System Inc.

Since the eigenvalues of the product XY are input-output invariant, a similarity transformation ($A_b = T^{-1}AT, B_b = T^{-1}B, C_b = CT$) can be performed to diagonalize the product XY as

$$T^{-1}XYT = \Sigma = diag({\sigma_1}^2, {\sigma_2}^2, \ldots, {\sigma_n}^2) \tag{3}$$

The eigenvalues, which characterize the importance of state variables in terms of energy transfer, are arranged in a descending order. If we partition the matrices as

$$\begin{bmatrix} W_1^T \\ W_2^T \end{bmatrix} XY \begin{bmatrix} V_1 & V_2 \end{bmatrix} = \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \tag{4}$$

in which Σ_1 contains r dominant eigenvalues, and the columns of W_1 and V_1 are corresponding left and right eigenvectors, respectively, a reduced model can be obtained as follows

$$\begin{array}{rcl} \dot{x}(t) & = & A_r x(t) + B_r u(t) \\ y(t) & = & C_r x(t) \end{array} \tag{5}$$

where $A_r = W_1^T A V_1$, $B_r = W_1^T B$, $C_r = CV_1$. The error in the transfer function of the order r approximation is bounded by $2\sum_{i=r+1}^{n} \sigma_k$.

III. SECOND-ORDER BALANCED TRUNCATION

Consider a second-order LTI stable system

$$\begin{array}{l} M\ddot{q}(t) + D\dot{q}(t) + Kq(t) = Bu(t) \\ y(t) = Pq(t) + Q\dot{q}(t) \end{array} \tag{6}$$

where $u(t) \in R^p$, $y(t) \in R^q$, $q(t) \in R^n$, $B \in R^{n\times p}$, $P, Q \in R^{q\times n}$, $M, D, K \in R^{n\times n}$ with M assumed to be nonsingular.

The general idea of reducing the second-order system is to transform the second-order system first into the equivalent first-order system, from which the balancing matrices are obtained. To this end, the second-order Gramians in [5] were defined based on the first-order realization in a standard state-space form (1) with *2n*-dimensional state $x^T = [q^T \dot{q}^T]$, where

$$\mathcal{A} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} 0 \\ M^{-1}B \end{bmatrix} \qquad \mathcal{C} = \begin{bmatrix} P & Q \end{bmatrix} \tag{7}$$

The first-order realization has the same input-output behavior as the second-order system. Although a first-order MOR approach, like classic balanced truncation [6], can be applied to reduce (7), the reduced model is not a second-order system anymore in general. To perform the reduction directly on the second-order equations (6), one needs to define Gramians for second-order systems. Similar to the first order Gramian definition, the second order Gramian definition is based on the following optimization problem [5]

$$\begin{array}{l} min_{\dot{q}(0)\in R^n, u\in L_2[-\infty,0]} \left(J = \int_{-\infty}^{0} u^T(t)u(t)dt\right) \\ \text{subject to} \\ M\ddot{q}(t) + D\dot{q}(t) + Kq(t) = Bu(t) \\ q(0) = q_0 \end{array} \tag{8}$$

which minimizes the necessary energy to reach the given q_0 over all past inputs and initial $\dot{q}$. If we compatibly partition the controllability Gramian of the first-order realization (7) as

$$\mathcal{X} = \begin{bmatrix} R & S \\ S^T & F \end{bmatrix} \tag{9}$$

then the optimum for the problem (8) is $q_0^T R^{-1} q_0$ and thus the controllability Gramian of the second-order system is $X_2 = R$. Similarly, if we compatibly partition the observability Gramian of the first-order realization (7) as

$$\mathcal{Y} = \begin{bmatrix} U & V \\ V^T & N \end{bmatrix} \tag{10}$$

then the observability Gramian of the second-order system is $Y_2 = U$. The eigenvalues of the Gramian product X_2Y_2 are invariant under a similarity transformation

$$T^{-1}X_2Y_2T = diag({\sigma_{21}}^2, {\sigma_{22}}^2, \ldots, {\sigma_{2n}}^2) \tag{11}$$

Similar to the first-order case, the matrices can be partitioned as

$$\begin{bmatrix} W_1^T \\ W_2^T \end{bmatrix} X_2Y_2 \begin{bmatrix} V_1 & V_2 \end{bmatrix} = \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \tag{12}$$

where Σ_1 contains the first r largest eigenvalues and W_1, V_1 are corresponding left and right eigenvectors, respectively. A reduced second-order model can be obtained as follows

$$\begin{array}{l} M_r\ddot{q}(t) + D_r\dot{q}(t) + K_r q(t) = B_r u(t) \\ y(t) = P_r q(t) + Q_r\dot{q}(t) \end{array} \tag{13}$$

in which

$$\begin{array}{l} M_r = W_1^T M V_1, D_r = W_1^T D V_1, K_r = W_1^T K V_1 \\ B_r = W_1^T B, P_r = PV_1, Q_r = QV_1 \end{array} \tag{14}$$

However, in order to preserve the symmetry and stability of the original system, an orthogonal projection is performed in [5] as follows

$$\begin{array}{l} M_r = V_1^T M V_1, D_r = V_1^T D V_1, K_r = V_1^T K V_1 \\ B_r = V_1^T B, P_r = PV_1, Q_r = QV_1 \end{array} \tag{15}$$

where the equations are left multiplied by V_1 instead of W_1. Unfortunately, since $W_1 \neq V_1$ for a non-symmetric system (7), the resulting Gramian product X_2Y_2 will not be balanced and the accuracy is sacrificed.

IV. THE SBPOR ALGORITHM

In this section, we introduce the new second-order balanced truncation method SBPOR and its Gramian approximation version.

A. Symmetric realization in descriptor form

As mentioned before, RLCK circuits can be formulated in a second-order form (6) with special structure $M = C$, $D = G$, $K = \Gamma$, $P = 0$, $Q = B^T$

$$\begin{array}{l} C\ddot{q}(t) + G\dot{q}(t) + \Gamma q(t) = Bu(t) \\ y(t) = B_2^T\dot{q}(t) \end{array} \tag{16}$$

where $u(t)$, $y(t) \in R^p$ are input currents and output voltages; $\dot{q}(t) \in R^n$ are nodal voltages; $G, C, \Gamma \in R^{n\times n}$ are matrices of conductance, capacitance and susceptance respectively and $C = C^T > 0, G = G^T \geq 0, \Gamma = \Gamma^T \geq 0$; $B \in R^{n\times p}$ is the input matrix and its transpose $B^T \in R^{p\times n}$ is the output matrix. Note that C is assumed to be invertible [4], [5].

The key idea in this paper is that instead of using the first-order realization (7), we choose another first-order realization in descriptor form [4] with *2n*-dimensional state $x^T = [q^T, \dot{q}^T]$

$$\begin{array}{l} \mathcal{E}\dot{x}(t) = \mathcal{A}x(t) + \mathcal{B}u(t) \\ y(t) = \mathcal{B}^T x(t) \end{array} \tag{17}$$

where

$$\mathcal{E} = \begin{bmatrix} -\Gamma & 0 \\ 0 & C \end{bmatrix}, \mathcal{A} = \begin{bmatrix} 0 & -\Gamma \\ -\Gamma & -G \end{bmatrix}, \mathcal{B} = \begin{bmatrix} 0 \\ B \end{bmatrix} \quad (18)$$

Note that, since C, G, Γ are all symmetric, it follows $\mathcal{A} = \mathcal{A}^T, \mathcal{E} = \mathcal{E}^T$, which means such a first-order realization is symmetric.

Controllability and observability Gramians in descriptor form can be computed from a pair of generalized Lyapunov equations [12]. However, in this symmetric case, both Gramians are equal and only one equation is to be solved

$$\mathcal{E}\mathcal{X}\mathcal{A}^T + \mathcal{A}\mathcal{X}\mathcal{E}^T + \mathcal{B}\mathcal{B}^T = 0 \quad (19)$$

If we compatibly partition the Gramians as

$$\mathcal{X} = \mathcal{Y} = \begin{bmatrix} R & S \\ S^T & F \end{bmatrix} \quad (20)$$

then the second-order Gramians are also equal

$$X_2 = Y_2 = R \quad (21)$$

Since Gramian is symmetric, R is orthogonally diagonalizable, i.e., there exists $T^{-1} = T^T$ such that

$$T^T R T = \Sigma \quad (22)$$

As a result, the second-order Gramian product RR can be orthogonally diagonalized as

$$T^T R R T = (T^T R T)(T^T R T) = (\Sigma)^2 \quad (23)$$

Note that the eigenspace of the Gramian product is exactly the eigenspace of each Gramian. If we partition the matrices in (22) as

$$\begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} R \begin{bmatrix} V_1 & V_2 \end{bmatrix} = \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \quad (24)$$

where Σ_1 contains the first r largest eigenvalues of Gramian R and V_1 are corresponding eigenvectors, a reduced model can be obtained as follows

$$\begin{aligned} &C_r \ddot{q}(t) + G_r \dot{q}(t) + \Gamma_r q(t) = B_r u(t) \\ &y = B_{2r}^T \dot{q}(t) \end{aligned} \quad (25)$$

where $C_r = V_1^T C V_1, G_r = V_1^T G V_1, \Gamma_r = V_1^T \Gamma V_1, B_r = V_1^T B$. This kind of transformation is known as congruency transformation, which preserves symmetry and definiteness of matrices such that $C_r = C_r^T \geq 0, G_r = G_r^T \geq 0, \Gamma_r = \Gamma_r^T \geq 0$, implying the reduced-order system has guaranteed stability, passivity, and reciprocity [13]. The basic algorithm flow for SBPOR is given in Fig. 1.

ALGORITHM 1: SBPOR
Input: C, G, Γ, B
Output: C_r, G_r, Γ_r, B_r

1) Form the symmetric first-order realization in descriptor form (17)
2) Solve $\mathcal{E}\mathcal{X}^T\mathcal{A}^T + \mathcal{A}\mathcal{X}\mathcal{E}^T + \mathcal{B}\mathcal{B}^T = 0$ for $\mathcal{X}$
3) Partition $\mathcal{X}$ as:
   $\mathcal{X} = \begin{bmatrix} R & S \\ S^T & F \end{bmatrix}$
4) Compute SVD of the second-order Gramian:
   $R = \begin{bmatrix} V_1 & V_2 \end{bmatrix} \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix}$
5) Form the reduced model as
   $C_r = V_1^T C V_1, G_r = V_1^T G V_1, \Gamma_r = V_1^T \Gamma V_1, B_r = V_1^T B$

Fig. 1. The SBPOR algorithm.

B. Second-order Gramian approximation

We also propose a second-order Gramian approximation technique to mitigate high computational cost. Practically, we find that Γ can easily become singular, which will make both $\mathcal{A}$ and $\mathcal{E}$ in (18) singular. To mitigate this problem, we propose a little different symmetric realization. If we define $x^T = [E_l^T q^T, \dot{q}^T]$, we have the following new realization:

$$\begin{aligned} &\mathcal{C}\dot{x}(t) = -\mathcal{G}x(t) + \mathcal{B}u(t) \\ &y(t) = \mathcal{B}^T x(t) \end{aligned} \quad (26)$$

where

$$\mathcal{C} = \begin{bmatrix} -L^{-1} & 0 \\ 0 & C \end{bmatrix}, \mathcal{G} = \begin{bmatrix} 0 & E_l L^{-1} \\ L^{-1} {E_l}^T & G \end{bmatrix}, \mathcal{B} = \begin{bmatrix} 0 \\ B \end{bmatrix} \quad (27)$$

Here E_l is the incidence matrix for inductor matrix L in the modified nodal analysis (MNA) formulation and $\Gamma = E_l L^{-1} {E_l}^T$. We remark that $E_l L^{-1}$ will not have zero rows for a physical system as $E_l q$ is actually a vector of a branch vector potential [19]. So $\mathcal{G}$ will not be singular, required by our new SOGA algorithm, for any physical system that has DC paths to ground for any node.

Since C, G, L are all symmetric, such a first-order realization is also symmetric and the second-order Gramian measures the contribution of the node voltages $\dot{q} = v$ with respect to the transfer function.

For first-order system in descriptor form (26), the Gramian $\mathcal{X}$ can be computed from the expression in frequency domain [12]

$$\mathcal{X} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} (j\omega\mathcal{C} + \mathcal{G})^{-1} \mathcal{B}\mathcal{B}^T (j\omega\mathcal{C} + \mathcal{G})^{-H} dw \quad (28)$$

Let ω_k be kth sampling point over the frequency range of interests. If we define

$$z_k = (j\omega_k \mathcal{C} + \mathcal{G}^{-1})\mathcal{B} \quad (29)$$

then $\hat{\mathcal{X}}$ can be approximately computed as

$$\hat{\mathcal{X}} = \frac{1}{2\pi} \sum z_k z_k^H = \mathcal{Z}\mathcal{Z}^H \quad (30)$$

where $\mathcal{Z}$ is a matrix whose columns are z_k. If we partition $\mathcal{Z}^H = \begin{bmatrix} Z_1^H & Z_2^H \end{bmatrix}$ and compatibly partition the approximated Gramian as

$$\hat{\mathcal{X}} = \begin{bmatrix} \hat{R} & \hat{S} \\ \hat{S}^T & \hat{F} \end{bmatrix} = \begin{bmatrix} Z_1 Z_1^H & Z_1 Z_2^H \\ Z_2 Z_1^H & Z_2 Z_2^H \end{bmatrix} \quad (31)$$

then the approximated second-order Gramian is $\hat{F}$, which can be diagonalized as

$$\begin{aligned} &\hat{F} = Z_2 Z_2^H = (\hat{U}\hat{\Sigma}\hat{V})(\hat{U}\hat{\Sigma}\hat{V})^T = \hat{U}\hat{\Sigma}^2\hat{U}^T = \\ &\begin{bmatrix} \hat{U}_1 & \hat{U}_2 \end{bmatrix} \begin{bmatrix} \hat{\Sigma}_1^2 & 0 \\ 0 & \hat{\Sigma}_2^2 \end{bmatrix} \begin{bmatrix} \hat{U}_1^T \\ \hat{U}_2^T \end{bmatrix} \end{aligned} \quad (32)$$

Therefore, $\hat{U}_1$ will be used to perform the reduction as in the SBPOR method. The SOGA algorithm is presented in Fig. 2.

V. EXPERIMENTAL RESULTS

In this section, we show examples that illustrate the effectiveness of proposed SBPOR method and compare it with existing relevant MOR approaches.

ALGORITHM 2: SOGA
Input: C, G, Γ, B
Output: C_r, G_r, Γ_r, B_r

1) Start from the symmetric first-order realization (26)
2) Do until satisfied:
3) Select a frequency points s_k
4) Compute $z_k = (s_k\mathcal{C} + \mathcal{G})^{-1}\mathcal{B}$
5) Form $\mathcal{Z}_k = [z_1, z_2, \ldots, z_k]$ and partition $\mathcal{Z}_k = \begin{bmatrix} Z_{k1} \\ Z_{k2} \end{bmatrix}$
6) Compute the SVD of the matrix Z_{k2}. If the error is satisfactory, go to Step 7. Otherwise, go to Step 2.
7) Form the projection matrix $\hat{U}_1$ from the singular vectors of Z_k, dropping ones corresponding to small singular values below a desired tolerance, and form the reduced model as
$C_r = \hat{U}_1^T C \hat{U}_1, G_r = \hat{U}_1^T G \hat{U}_1, \Gamma_r = \hat{U}_1^T \Gamma \hat{U}_1, B_r = \hat{U}_1^T B$

Fig. 2. The SOGA algorithm.

A. Comparison with first-order TBR

Given a circuit in the form (16), we first compare SBPOR with the first-order TBR method. Note that the order q in the reduced models reduced by SBPOR on (16) will correspond to the order of $2q$ in the reduced models by the first-order TBR method performed on equivalent first-order realization (7). We choose a small circuit for the purpose of illustration so that both impedances and real parts can be compared at all possible reduced orders. The RLCK circuit has 4 nodal voltages and thus has a dimension of 4 in a second-order formulation. The equivalent first-order realization has a dimension of 8. As shown in Fig. 3(a),(b),(c), SBPOR outperforms standard

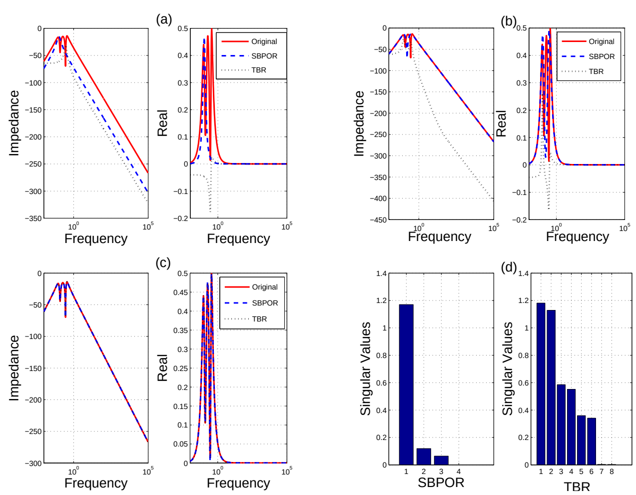

Fig. 3. Comparison with the first-order TBR method (performed on linearied first-order system).

TBR at each reduced order (q=1,2,3). This can be explained from the 'energy' distribution of singular values as shown in Fig. 3(d), where the second-order singular values decay much faster than the first-order ones. The passivity of reduced models can be tested from the real parts. As expected, SBPOR can guarantee the passivity of reduced models while standard TBR cannot. As shown in Fig. 3(a),(b),(c), only in Fig. 3(c), the real part of TBR reduced model is positive at all frequencies and thus the reduced model is passive. Note that standard TBR applied to the equivalent first-order realization(7) also results in a first-order reduced model and thus is not a second-order MOR approach available. We just use it as a criterion to show the accuracy of our new approach.

B. Comparison with SAPOR

In the second example, we want to compare our method with moment-matching based second-order MOR approach SAPOR [13]. The example is an RLCK circuit, which has 100 nodal voltages. The reduced second-order model has a dimension of 2. As shown in Fig. 4(a), SBPOR is globally

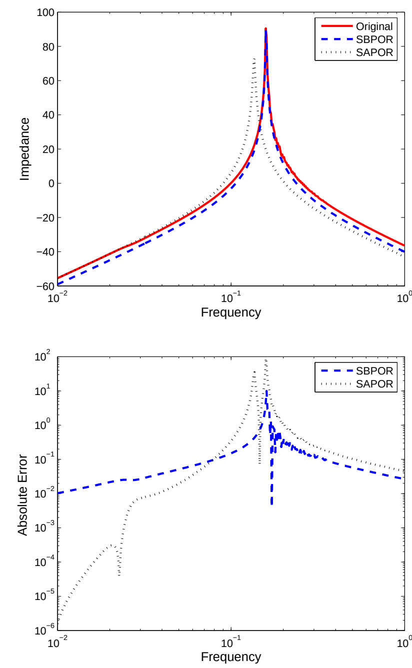

Fig. 4. Comparison with Krylov-based second-order MOR method SAPOR [13].

accurate at all frequencies while SAPOR has very good local behavior around DC (the expansion point of SAPOR is 0.01 Hz) but behaves so bad at other frequencies. The error is shown in Fig. 4(b), where the maximum absolute error for SBPOR is about 10 but for SAPOR is almost 100.

C. Comparison with existing second-order TBR

In this part, we want to compare the new method, SBPOR, with existing technique [5] in the control literature, which we name TBR2. The example is an RLCK circuit with 100 nodal voltages and the reduced dimension is 10. In Fig. 5(a), we can see that SBPOR outperforms TBR2 obviously. As shown in Fig. 5(b), the maximum absolute error for SBPOR is smaller than 10 while it is almost 100 for TBR2. The reason is that the system in TBR2 is not really balanced and thus the accuracy is sacrificed.

D. Comparison of SOGA with SAPOR

The original model is an RLCK circuit with 1000 nodes in a second-order formulation. The reduced model has an order of 11 ($q = 11$). As shown in Fig. 6, SOGA produces a better approximation than SAPOR over a wide frequency band (the expansion point of SAPOR is 1 Hz). The computational cost of SOGA is almost the same as that of SAPOR given the same reduction order. The reduction CPU times of several mesh-structured RLC examples are shown in Table I, where the n is the number of nodes and the reduced order is 10.

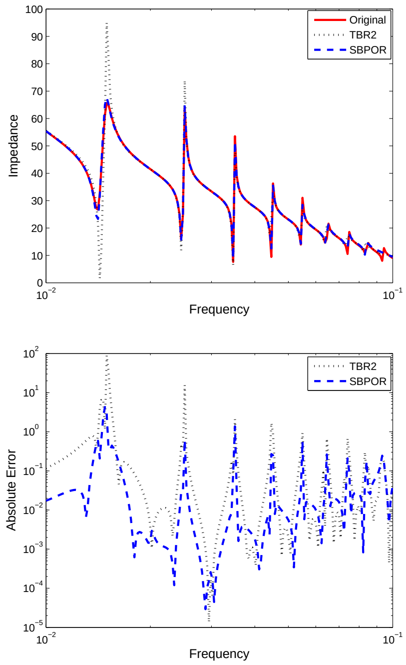

Fig. 5. Comparison with the existing second-order TBR method [5].

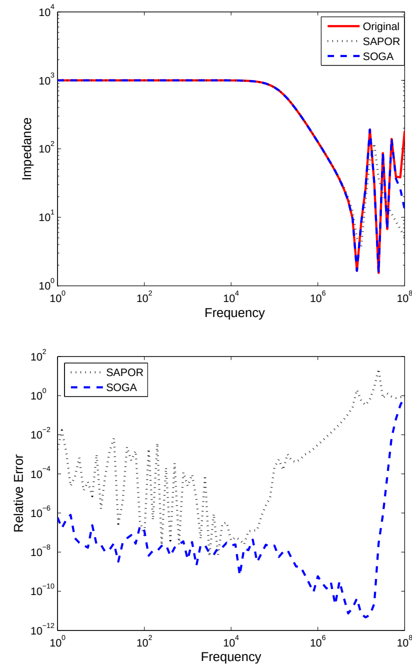

Fig. 6. Accuracy comparison between SOGA and SAPOR.

TABLE I

REDUCTION CPU TIME COMPARISON OF SOGA AND SAPOR (SECONDS).

	n=640	n=1000	n=2680	n=4380
SOGA	3.257	6.875	25.24	602.63
SAPOR	1.438	3.420	21.42	580.57

VI. CONCLUSION

In this paper, we have proposed a novel method, SBPOR and its faster Gramian computation version, SOGA, for the model order reduction of RLCK circuits. SBPOR utilizes a symmetric first-order realization in descriptor form so that the second-order system can be really balanced via congruency transformation without any accuracy loss, which in contrast with the existing second-order balanced truncation [5]. Experimental results show that SBPOR is more accurate than existing second-order balanced truncation method [5]. SOGA is also globally more accurate than the second-order Krylov subspace based method SAPOR [13] with similar computational cost.

REFERENCES

[1] A. Devgan, H. Ji, and W. Dai, "How to efficiently capture on-chip inductance effects: introducing a new circuit element K," in *Proc. Int. Conf. on Computer Aided Design (ICCAD)*, 2000, pp. 150–155.

[2] Z. He, M. Celik, and L. Pillegi, "SPIE: Sparse partial inductance extraction," in *Proc. Design Automation Conf. (DAC)*, 1997, pp. 137–140.

[3] H. Ji, A. Devgan, and W. Dai, "Ksim: A stable and efficient RKC simulator for capturing on-chip inductance effect," in *Proc. Asia South Pacific Design Automation Conf. (ASPDAC)*, 2001, pp. 379–384.

[4] A. J. Laub and W. F. Arnold, "Controllability and observability criteria for multivariable linear second-order models," *IEEE Trans. Automat. Contr.*, vol. 29, pp. 163–165, 1987.

[5] D. G. Meyer and S. Srinivasan, "Balancing and model reduction for second-order form linear systems," *IEEE Trans. Automat. Contr.*, vol. AC-41, pp. 1632–1644, 1996.

[6] B. Moore, "Principle component analysis in linear systems: Controllability, and observability, and model reduction," *IEEE Trans. Automat. Contr.*, vol. 26, no. 1, pp. 17–32, 1981.

[7] A. Odabasioglu, M. Celik, and L. Pileggi, "PRIMA: Passive reduced-order interconnect macromodeling algorithm," *IEEE Trans. on Computer-Aided Design of Integrated Circuits and Systems*, pp. 645–654, 1998.

[8] J. R. Phillips, L. Daniel, and L. M. Silveira, "Guaranteed passive balanced transformation for model order reduction," *IEEE Trans. on Computer-Aided Design of Integrated Circuits and Systems*, vol. 22, no. 8, pp. 1027–1041, 2003.

[9] J. R. Phillips and L. M. Silveira, "Poor man's TBR: a simple model reduction scheme," *IEEE Trans. on Computer-Aided Design of Integrated Circuits and Systems*, vol. 24, no. 1, pp. 43– 55, 2005.

[10] Z. Qi, H. Yu, P. Liu, S. X.-D. Tan, and L. He, "Wideband passive multiport model order reduction and realization of RLCM circuits," *IEEE Trans. on Computer-Aided Design of Integrated Circuits and Systems*, pp. 1496–1509, Aug. 2006.

[11] B. N. Sheehan, "ENOR: model order reduction of RLC circuits using nodal equations for efficient factorization," in *Proc. Design Automation Conf. (DAC)*, 1999, pp. 17–21.

[12] L. M. Silveira and J. R. Phillips, "Exploiting input information in a model reduction algorithm for massively coupled parasitic networks," in *Proc. Design Automation Conf. (DAC)*, 2004, pp. 385–388.

[13] Y. Su, J. Wang, X. Zeng, Z. Bai, C. Chiang, and D. Zhou, "SAPOR: second-order Arnoldi method for passive order reduction of RCS circuits," in *Proc. Int. Conf. on Computer Aided Design (ICCAD)*, 2004, pp. 74–79.

[14] S. X.-D. Tan, "A general hierarchical circuit modeling and simulation algorithm," *IEEE Trans. on Computer-Aided Design of Integrated Circuits and Systems*, pp. 418–434, April 2005.

[15] S. X.-D. Tan and L. He, *Advanced Model Order Reduction Techniques in VLSI Design*. Cambridge University Press, 2007.

[16] N. Wang and V. Balakrishnan, "Fast balanced stochastic truncation via a quadratic extension of the alternating direction implicit iteration," in *Proc. Int. Conf. on Computer Aided Design (ICCAD)*, 2005, pp. 801–805.

[17] N. Wang, V. Balakrishnan, and C.-K. Koh, "Passivity-preserving model reduction via a computationally efficient projection-and-balance scheme," in *Proc. Design Automation Conf. (DAC)*, 2004, pp. 369–374.

[18] B. Yan, S. X.-D. Tan, P. Liu, and B. McGaughy, "Passive interconnect macromodeling via balanced truncation of linear systems in descriptor form," in *Proc. Asia South Pacific Design Automation Conf. (ASPDAC)*, 2007, pp. 355–360.

[19] H. Yu, Y. Shi, L. He, and D. Smart, "A fast block structure preserving model order reduction in inverse inductance circuits," in *Proc. Int. Conf. on Computer Aided Design (ICCAD)*, Nov. 2006, pp. 7–11.