Hurwitz Stable Model Reduction for Non-Tree Structured RLCK Circuits

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ABSTRACT

This paper presents an efficient way to compute the approximate time domain signal waveforms for RLCK circuits that have non-tree or tree-like structures. The new method is based on a graph based approach to drive transfer function of any linear circuits. Our contribution is the introduction of Hurwitz approximation to the truncated transfer functions to enforce the stability of reduced systems. We also extend the direct truncation of the transfer (DTT) technique , which can only work for tree-structured circuits, to deal with non-tree or tree-like RLC circuits. By combining DTT technique with graph-based method, we show that the new method is capable of analyzing non-tree or tree-like structured RLCK circuits which are more accurate models of deep submicron high-speed coupled interconnects. The proposed method has been tested and validated on some coupled RLCK circuits.

I. INTRODUCTION

As feature size keeps shrinking and clock rate continues roaring, interconnect related signal integrity issues become more severe[1]. Fast and accurate evaluation and modeling of interconnect networks' behavior is critical for interconnect-centered physical design and optimization paradigm for SoC systems [5, 6].

A number of model-order reduction based techniques have been introduced [2, 3, 4, 9, 10, 13, 14] to characterize the transient behavior of interconnects. Asymptotic Waveform Evaluation (AWE) algorithm was first proposed [10] where explicit moment matching is used to compute the dominant poles at low frequency. But AWE method is numerically unstable for higher order approximation. Thereafter a number of other model-order reduction methods based on implicit moment matching (via Krylov subspace projection) were developed. Examples are Pade via Lanczos (PVL) [2], Matrix PVL [3], Arnoldi method [13], Arnoldi Transformation method [14], Passive reduced-order interconnect macromodeling algorithm (PRIMA) [9] and SyPVL algorithm [4]. The complexities of those algorithms, however, are higher than that of the AWE algorithm which is $O(nq)$ for RLC trees, n is the order of RLC tree and q is the reduced order of the tree.

Recently, direct truncation of the transfer function algorithm [8] was proposed for analyzing tree structured RLC interconnects. DTT has $O(nq^2)$ time complexity for general RLC trees and $O(nq)$ for binary trees. But DTT method can't obtain stable transfer functions for arbitrary high-order approximation. In this paper, we propose to apply Hurwitz approximation to obtain the stable truncated transfer functions for arbitrary high order approximation. Our work is inspired by recent approaches to model-order reduction of RLC tree circuits based on Hurwitz approximation [16].

Another drawback of DTT method is that it can only be applied to RLC tree circuits. In this paper, we extend DTT method to deal with non-tree or tree-like structured RLC circuits. Such non-tree structured RLC circuits are good models of general high-speed interconnects with both capacitive and inductive couplings among nodes in themselves and with their neighbor interconnects. As crosstalk due to capacitive and inductive couplings become dominant limiting factors for circuit performance, interconnects have to be modeled with non-tree structured dumped or distributed RCLK circuits and to be analyzed efficiently.

In this paper, we propose a new approach to analyze non-tree structured RLCK circuits. The new approach is based on a newly proposed graph based approach to drive the transfer functions of any linear circuits [11, 12]. The new graph used in our method is called determinant decision diagrams (DDDs). DDD graphs was successfully used for symbolic analog circuit analysis [11, 12]. In this paper, we extend this technique to analyze RLCK linear circuits. For a large interconnect net that connects one driver and many sinks, its topology is still a tree without considering coupling effects. For such tree-like structured coupled RLCK circuits, we combined both DTT method with our graph based approach to efficiently compute the model-reduced transfer functions between the source to any sink node. The resulting algorithm can deal with linear circuits with *any structures* and is more efficient for tree-like structured linear circuits due to efficiency of DTT algorithm.

This paper is organized as follows. Section II reviews the concepts of DTT algorithm and Hurwitz approximation. In section III, we first explain the concepts of DDDs and s-expanded DDDs for driving transfer functions of linear networks, then we present the new method for generating truncated transfer functions for tree-like structured RLC networks. Experimental results are described in Section IV. Section V concludes the paper.

II. REVIEW OF DTT AND HURWITZ APPROXIMATION

A. DTT Algorithm

DTT method is based on the fact that transfer functions of a node in a RLC tree circuit can be generated from the transfer functions of its downstream subcircuits in the tree circuit. An RLC tree circuit is shown in Fig. 1, where a basic RLC section consists of R_1 and L_1 and C_1 and this RLC section is cascaded and repeated in the left and the right subcircuits recursively. DTT algorithm exploits the fact that for a passive RLC tree circuit, the poles of the circuit in its voltage gain transfer function are the zeros of the impedance seen at the input of the circuit. As a result, the poles of the RLC circuit are also the zeros of transfer functions at the same node and all the nodes in its parallel branches in the whole circuit where the RLC network belongs to. For instance in Fig. 1, if we obtain the poles of the left subcircuit (when it is disconnected from the whole circuit), the poles will become the zeros of the transfer functions at node 1 and all the other nodes in right subcircuit in the whole circuit. In terms of transfer function construction, this means that we can compute the numerators of transfer function at a node when we know the denominators of all its downstream subcircuits (when they are disconnected). In the following we refer to this rule as *pole-zero relationship rule*. We notice that such a bottom-up transfer construction strategy is generally possible only for a tree-structured circuit.

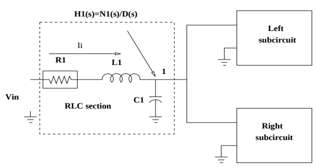

Fig. 1. Transfer function construction rules in DTT

In frequency domain, let $H_i(s) = \frac{V_i}{V_{in}} = \frac{N_i(s)}{D(s)}$, we have

$$1 - H_1(s) = (sR_1 + s^2L_1)\sum_j C_j H_j(s), \quad (1)$$

$$D(s) - N_1(s) = (sR_1 + s^2L_1)\sum_j C_j N_j(s). \quad (2)$$

According to the pole-zero relationship rule, we have

$$N_1(s) = D_l(s) \times D_r(s), \quad (3)$$

where $D_l(s)$ and $D_r(s)$ are denominators from left and right subcircuits in Fig. 1 respectively. From (2), we have

$$D(s) = N_1(s) + (sR_1 + s^2L_1)M_1, \quad (4)$$

$$M_1 = C_1N_1(s) + M_l(s) \times D_r(s) + M_r(s) \times D_l(s). \quad (5)$$

With equations (3), (4) and (5), we can recursively compute the denominators at every node (when the node is disconnected from the rest of the circuit) in a bottom up fashion. Once the denominator is obtained for node 1 in Fig. 1, which is also the denominator for all the other nodes in the downstream circuit, numerators at other downstream nodes are computed in a top-down fashion by using the pole-zero relationship rule. During the process, any higher order (say larger than q) coefficients are discarded.

Once the reduced order transfer function $H_q(s)$ is determined, it can be transformed by efficient numerical methods into the following partial fraction form:

$$H_q(s) = \frac{k_1}{s-p_1} + \frac{k_2}{s-p_2} + \ldots + \frac{k_q}{s-p_q}. \quad (6)$$

The time domain response of some input signals can be obtained by taking the reverse Laplace transform of $\frac{k_i}{s-p_i}V_{in}(s)$ for each $\frac{k_i}{s-p_i}$ item, where, $V_{in}(s)$ is the input signal.

B. Hurwitz Approximation

As stated in [16], Hurwitz approximation is an effective method to enforce the stability of the order-reduced transfer functions of the RLC circuit system. A polynomial is called a *Hurwitz polynomial* if all its roots have negative real parts [15]. For a given RLC system with a rational transfer function $H(s)$. $H(s)$ is stable if and only if its denominator is a Hurwitz polynomial [15].

Suppose the transfer function for a RLC system is

$$H(s) = \frac{N(s)}{D(s)} = \frac{1 + a_1s + a_2s^2 + \ldots + a_ps^p}{b_0 + b_1s + b_2s^2 + \ldots + b_qs^q}, \quad (7)$$

Without losing generality, let q be even. We use $m(s)$ and $n(s)$ to represent the even and odd parts of $D(s)$ respectively:

$$m(s) = b_0 + b_2s^2 + \ldots + b_qs^q, \quad (8)$$

$$n(s) = b_1 + b_3s^3 + \ldots + b_{q-1}s^{q-1}. \quad (9)$$

Then the fraction $n(s)/m(s)$ can be written as the following form:

$$\frac{n(s)}{m(s)} = \frac{1}{c_0s^{-1} + \frac{1}{c_1s^{-1} + \frac{1}{\ldots + \frac{1}{c_{q-1}s^{-1}}}}}, \quad (10)$$

where $D(s)$ is said to be a Hurwitz polynomial if the coefficients c_0, c_1, . . .,c_{q-1} are all positive. To find a kth-order Hurwitz approximation $H_k(s)$ for a given transfer function $H(s)$, we can keep all the positive coefficients c_0 up to c_k and ignore all the other coefficients, and then re-calculate the odd and even parts of the denominator $D_k(s)$ of $H_k(s)$ according to equation (10). Once we obtain the stable new denominator $D_k(s)$, the new numerator, $N_k(s)$, in the final approximated rational function

$$H_k(s) = \frac{N_k(s)}{D_k(s)}, \quad (11)$$

can be obtained with explicit moment matching by which the first k moments are equivalent in $H_k(s)$ and $H(s)$, or $H(s) - H_k(s) = O(s^k)$. The new transfer function $H_k(s)$ is the kth-order Hurwitz approximation of the original system and is guaranteed to be stable. Note that the explicit moment matching here is numerical stable for very high orders as no ill-conditioned matrix is solved as is the case for AWE [10].

III. NEW DDD-BASED DTT ALGORITHM

Before we present our new algorithm, we first provide a brief overview of the notion of determinant decision diagrams [11]. We review how a s-expanded DDD can be used to drive exact transfer functions of a linear circuit.

A. Generation of Transfer function via Determinant Decision Diagrams

Determinant Decision Diagrams [11] are compact and canonical graph-based representation of determinants. A DDD is a signed, rooted, directed acyclic graph with two terminal vertices, namely the *0-terminal* vertex and the *1-terminal* vertex. Each non-terminal DDD vertex is labeled by a symbol in the determinant denoted by a_i, and a positive or negative sign denoted by $s(a_i)$. It *originates* two outgoing edges, called *1-edge* and *0-edge*. Each vertex a_i represents a symbolic expression $D(a_i)$

defined recursively as follows: $D(a_i) = a_i \; s(a_i) \; D_{a_i} + D_{\overline{a}_i}$, where D_{a_i} and $D_{\overline{a}_i}$ represent, respectively, the symbolic expressions of the vertices pointed by the 1-edge and 0-edge of a_i. The 1-terminal vertex represents expression 1, whereas the 0-terminal vertex represents expression 0.

To exploit the DDD to derive transfer functions, we need to directly represent circuit parameters not matrix entries. To this end, s-expanded DDDs are introduced [12]. The s-expanded DDD can be constructed from the complex DDD linearly in the size of the original complex DDD [12]. Once a coefficient DDD is constructed, its numerical value can be easily computed by simply traversing all the vertices in the coefficient DDD graph and performing one addition and one multiplication at each vertex [11]. In this way, we can derive the transfer functions of any linear circuits. Also DDD based method is numerically stable for very high order transfer function computation as only addition and multiplication are involved.

B. New DTT Method for Non-Tree or Tree-like Structured RLC Circuits

For a general RLCK circuit, which does not have any repeated regular structures, DTT method can't be applied directly as the method relies on the exploitation of tree structures. DDD-based method, however, can drive transfer functions for linear circuits with any structures. But for very large circuits, DDD-based method may not be very efficient unless the circuit is a ladder-structured circuit [11].

On the other hand, real interconnect nets typically bear tree-like structures as mentioned early. As a result, we combine both DTT method and DDD-based method to analyze such RLCK circuits. So in the following, we focus on the RLCK circuits with tree-like structures where coupling loops may exist at or near leaf nodes. So all the terminologies for tree are still used. We first show that the pole-zero relationship rule defined in the subsection II is still valid for a general passive RLC network when only one input port of the circuit is externally excited. We have the following result without proof.

Proposition 1 *For a passive RLC circuit which is excited at only one input of the circuit, the poles of the circuit voltage transfer function are zeros of the impedance seen at the input port of the circuit.*

As a result, for any subcircuit, which can have any structure and has only one input port driven by its parent circuit, the recursive Eq. (3), (4) and (5) are still valid. In other words we are still able to compute the transfer functions at any upstream nodes in the RLC parent circuit using those equations. Transfer functions (specifically the numerators) at nodes inside such subcircuits can also be computed by using the pole-zero relationship rule as the rule applies to any linear circuit. In the sequel, such a non-tree structured subcircuit, which does not have any connection with other circuits except at the input of the circuit is called a *composite* subcircuit.

The last task left is to compute the transfer functions of the composite subcircuits when they are disconnected from the whole circuit. This is done by the DDD-based transfer-function computation technique, which is efficient for any medium sized linear circuits. In the next subsection, we detail the new modified DTT algorithm flow.

C. New Modified DTT Algorithm Flow

The first thing in our analysis scheme is to identify those coupled subcircuits and combine them into one subcircuit such that the resulting RLC circuit becomes a tree again and the traditional DTT method can still be applied. The new method is called DDD-based DTT, or $\mathbf{D^2TT}$ method. The new algorithm is outlined in the following:

DDD-based DTT ($\mathbf{D^2TT}$) Algorithm

1. Group all the subcircuits, which have common couplings, into a single composite subcircuit in a bottom up fashion.
2. For each composite subcircuit, DDD-based algorithm is used to compute the voltage gain transfer functions (numerators and denominators) at all the nodes inside those subcircuits.
3. Use Eq. (3), (4) and (5) to compute all the other nodes in the circuit in a bottom up way until we obtain the denominator of the whole circuit. At the same time, higher order terms are trimmed at every step.
4. Compute the numerator at every node in a top-down fashion using the zero-pole relationship rule. All the nodes in the composite subcircuits are also computed in the same way as the numerator at each node of the disconnected subcircuits is already computed by DDD-based method.
5. Perform the Hurwitz approximation to obtain a Hurwitz stable transfer function.

Once the reduced order transfer functions are determined, we proceed to compute the poles and residues in the partial fraction form as traditional DTT method does.

The time complexity of DDD-based transfer function computation is proportional to the number of DDD vertices used. If the underlying circuit is a ladder circuit, DDD-based algorithm is linear in sizes of circuits. For general circuits, the size of DDD graph may grow exponentially in the worse case. But like BDDs, with proper vertex ordering, the DDD representations are very compact for many real circuits [11, 12]. The time complexity of the new $\mathbf{D^2TT}$ method is practically close to O(nq).

IV. Experiment Results and Discussion

The new $\mathbf{D^2TT}$ algorithm is implemented in C++ and has been tested on a number of coupled RLC circuits. In this section we present the experimental results on one of such coupled RLC circuits shown in Fig. 2. In the circuit, both capacitive and inductive coupling loops exist in some subcircuits and the RLC circuit has two composite subcircuits. Since L_4 couples with L_5, so all the subcircuits from node 7 downstream become one composite subcircuit. Another composite subcircuit is from node 19 downstream. The two composite subcircuits are marked within dotted lines in the figure. After the reduced order transfer functions are obtained, *Matlab 6.1* is used to obtain the poles and residues and then to compute the time domain responses with different input signals.

In Fig. 3, the transient step responses at node 13 are obtained for different order Hurwitz approximations. The input ramp signal is $V_{in}(s) = \frac{1}{T_t s^2}$, where $T_t = 100fs$. The exact response is also plotted in each case for comparison. We find that most poles are complex poles for RLC circuits.

We notice that some standard mathematic packages like *Maple-V* [7] and recent versions of *Matlab* also have symbolic transfer function computation capabilities, but DDD-based method is far more efficient and powerful than those methods as demonstrated in [11]. Some numerical curve fitting techniques can also be used to compute the transfer functions, but the resulting coefficients are approximate and may be quite different from the real coefficients which can lead to dramatically different poles.

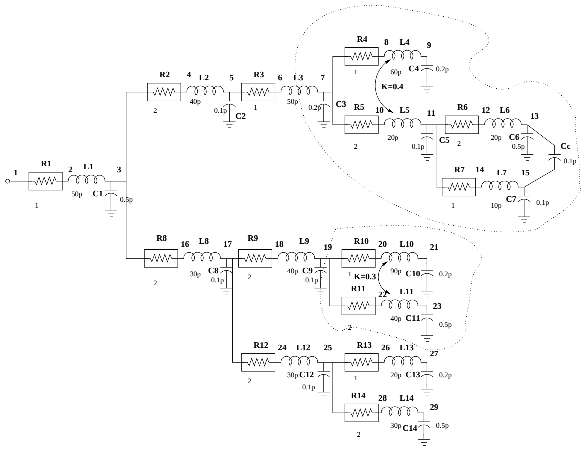

Fig. 2. A RLC circuits with both capacitive and inductive couplings

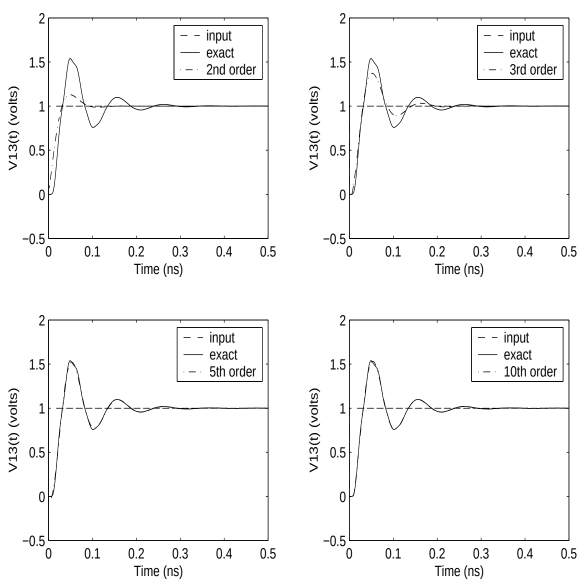

Fig. 3. The ramp responses at node 13 for different order Hurwitz approximations

V. Conclusion

In this paper, a new model order reduction method has been proposed to characterize non-tree or tree-like structured RLC circuits. The new method is based on a graph based approach to drive transfer function of any linear circuits. By combining direct truncation of the transfer (DTT) technique with graph-based method, we show that the new method is capable of analyzing non-tree or tree-like structured RLCK circuits which are more accurate models of deep submicron high-speed coupled interconnects. Hurwitz approximation is performed on the truncated transfer function to enforce the stability of reduced system. Experimental results on a coupled RLC circuit has been provided. The resulting algorithms is superior to the original DTT algorithms due to enforced stability and added flexibility to handle more complicated interconnect structures due to capacitive and inductive coupling effects.

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