Finite Difference Method for Electromigration Analysis of Multi-Branch Interconnects

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***Abstract*—Electromigration (EM) in VLSI chips has become a major reliability issues in nanometer VLSI design. Traditional compact EM models cannot give accurate predictions about the stress evolution over all stress conditions for complicated multi-branch interconnect structures. In this paper, we try to mitigate this problem by performing finite difference method (FDM) for the EM effects in multi-branch interconnects based on the kinetics of the first principle of EM physics. We start with the partial differential equations that describe the fundamental hydrostatic stress evolution for both the void nucleation and void growth phases with proper boundary and initial conditions for typical multi-branch metal wires: the single 2-terminal wire, and the straight-line 3-terminal wires. The new FDM for EM analysis approach can easily accommodate existing non-uniformly distributed residual stress, while existing compact EM models cannot. Time varying temperature and current, which are also difficult to model with existing methods, can also be considered with this method. Numerical results show that the proposed FDM EM analysis method agrees with the COMSOL based finite element method in terms of accuracy.**

***Index Terms*—Electromigration, finite difference, interconnect, reliability**

I. Introduction

Electromigration (EM) in VLSI chips has become a major reliability issue in nanometer VLSI design. For the signoff of EM reliability, compact EM models such as Blacks equation [1] and Blech limit [2] can lead to over-design and 2x-3x larger guard bands [3]. Such conservative over-design rules, while already negatively affecting chip designs, will prove increasingly detrimental as chip timing requirements will be more difficult to meet with excessively large guard bands. Thus, a worst-case design methodology results in inefficiency and considerable penalties in chip area, performance, power, and reliability budgets. Consequently, more accurate EM modeling and analysis methods are necessary for efficient chip design.

Traditional compact EM models are unable to provide accurate stress evolution predictions over all stress conditions for complicated interconnect structures. Interconnect power grid networks rarely comprise of just straight-line wires and the EM effects on those interconnects are not independently affected [4] [5]. Currently, the Blech limit and Blacks equation are mainly modeled and analyzed empirically. The major drawbacks of those models are that they are not physics based and thus become less predictable over different stressed conditions while also being difficult to apply to more complicated wire structures (although some significant progress has been made in EM modeling recently).

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Recently, a compact physics-based EM model has been proposed for full-chip reliability analysis [6] [7] and is based on the hydrostatic stress diffusion equation. The new EM model has been extended to deal with multiple branch tree wires based on projected steadystate stress. A new analytical model for multi-branch interconnect trees has further been proposed which can accurately deliver transient stress evolution [8]. However, those solutions can be difficult, or even impossible, to extend to general interconnect tress with arbitrary structures as it is difficult to obtain the analytical solutions to those problems. Additionally, those models cannot accommodate existing residual stress (coming from stress migrations for instance) in general, which has significant impacts on the EM effects of wires.

In this work, we propose a new EM analysis method in which we perform the Finite Difference Method (FDM) on the EM effects in multi-branch interconnects based on the kinetics of the first principle of EM physics. We start with the basic and fundamental hydrostatic stress evolution partial differential equations (PDEs) for both the void nucleation and void growth phases with proper boundary and initial conditions for typical multi-branch metal wires: the single 2-terminal wire and the strait-line 3-terminal wires. The new FDM EM analysis method can easily accommodate the existing non-uniformly distributed residual stress, which cannot be done so in existing compact EM models. It can also consider transient thermal and applied current effects, which are also difficult to accommodate in existing compact EM models. By using a numerical method, we open the door for future speed up of the EM validation process by using more efficient simulation and modeling techniques. Numerical results show that the proposed FDM EM analysis method agrees with the COMSOL based finite element method In terms of accuracy.

II. EM Physics and modeling

EM is the physical phenomenon of the migration of metal atoms along the directions of applied electrical field. Atoms (either lattice atoms or defects/impurities) migrate toward the anode end of the metal wire along the trajectory of conducting electrons. During the migration process, hydrostatic stress will be generated inside the embedded metal wire due to momentum exchange between lattice arms and conduction electrons and is a prime cause of void and hillock formations at either end of the wire. indeed, when a metal wire is embedded into a rigid confinement, which is the case with interconnect metallization, the wire volume changes (induced by the atom depletion and accumulation due to migration) create tension at the cathode end and compression at the anode end of the line. Over time, the lasting unidirectional electrical load will increase hydrostatic stress, as well as the stress gradient which acts as counter-forces for atom migration along the metal line.

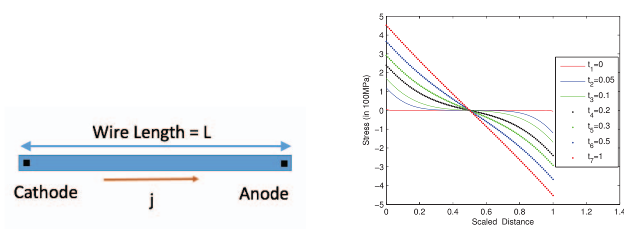
In some cases, usually when a line is long, this stress can reach a critical level, resulting in a void nucleation at the cathode and/or hillock formation at the anode end of the line. Existing Blacks model [1] is a semi-empirical model with consideration of physics. For instance, it does not consider the impact that residual stress or wire length have on the Mean Time to Failure (MTTF) of the wire. Also, when the wire reaches the MTTF, the wire is treated as an open circuit, which will over-estimate the EM-impacts in circuit reliability. Because of this, a new physics-based EM analysis method is used in this paper.

A. Void nucleation phase

Development of analytical formulation for electronic migration was proposed by Korhonen [9]. The failure process consists of two phases, nucleation and growth. In the first phase, void nucleation happens near the cathode edge of the line. Stress field $\sigma(x,t)$ is calculated by equation:

$$\begin{aligned} PDE &: \frac{\partial \sigma}{\partial t} = \frac{\partial}{\partial x}\left[\kappa\left(\frac{\partial \sigma}{\partial x}+G\right)\right], \text{ at } 0<t<t_{nuc} \\ BC &: \frac{\partial \sigma}{\partial x}(0,t) = G, \text{ at } 0<t<t_{nuc} \\ BC &: \frac{\partial \sigma}{\partial x}(L,t) = -G, \text{ at } 0<t<t_{nuc} \end{aligned} \quad (1)$$

Here, $\kappa = D_a B\Omega/kT$, where$D_a = D_0 exp(Ea/kT)$is the effective atomic diffusivity, E_a is the activation energy of the failure process, T is the absolute temperature and k is the Boltzmann constant. B is the effective bulk elasticity modulus, Ω is the atomic lattice volume, And $G = \frac{eZ\rho j}{\Omega}$, where e is the electron charge, eZ is the effective charge of the migrating atoms, ρ is the wire electrical resistivity, j is current density, x is the coordinate along the line, and t is time. The initial condition (IC) is the stress on the wire at time $t = 0$.For the boundary condition (BC), since the flux is blocked at both ends $x = 0$ and $x = L$ so $J(0,t) = J(L,t) = 0$ where$J(x,t) = \frac{D_a}{kT}(\frac{d\sigma(x,t)}{dx} + G)$



(a) (b)

Fig. 1. (a) Single wire with wire length L (b) EM-stress distribution change over time in a simple metal wire.

Fig. 1(a) shows a single wire with normalized length. Fig. 1(b) shows the EM-induced transient stress development for a single metal wire from the finite element analysis for a given current density and temperature setting. During this process, tensile stress (positive stress) will be developed at the cathode side (the left node), while compressive stress (negative stress) will be generated at the anode side of the wire (right node). When the tensile stress hits critical stress(σ_{crit}), a void will be generated at the cathode edge of line ($x = 0$). The time at which stress reaches σ_{crit} is called nucleation time(t_{nuc}).

B. Void growth phase

Void size growth begins in the second phase. After the nucleation phase, with growth of the void, resistance of the wires will be increased until growth saturates. The effective thickness of the void interface (δ) which is infinitely small in comparison with all other involved lengths is introduced. It allows us to introduce a stress gradient between the zero stress void surface and the surrounding metal as $\nabla\sigma = \sigma(\delta,t)/\delta$, where $\sigma(\delta,t) \approx \sigma(0,t)$ is the time dependent stress in the metal near the void surface. The differential condition is written as, [10]:

$$\begin{aligned} PDE &: \frac{\partial \sigma}{\partial t} = \kappa\frac{\partial^2 \sigma}{\partial x^2}, \text{ at } 0<t<\infty \\ BC &: \frac{\partial \sigma}{\partial x}(0,t) = \frac{\sigma(0,t)}{\delta}, \text{ at } 0<t<\infty \\ BC &: \frac{\partial \sigma}{\partial x}(L,t) = -G, \text{ at } 0<t<\infty \end{aligned} \quad (2)$$

IC is the stress at t_{nuc} in the nucleation phase since the growth phase starts immediately after nucleation which means $\sigma(x=0,t=0) = \sigma_{crit}$

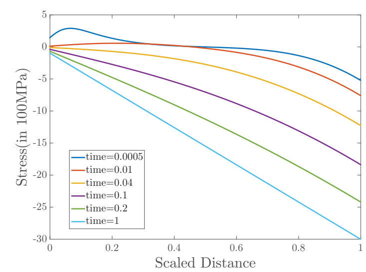


Fig. 2. EM-stress distribution change over time in simple metal wire for void growth.

Fig. 2 the evolution of stress starting from the void nucleation time $t = 0$ till the steady state is achieved.

III. Finite Difference Method for EM Analysis

The finite difference method (FDM) is a method of finding a numerical solution to partial differential equations (PDEs) [11]. Numerical methods allow us to find an approximate solution to a problem containing complex PDEs in which an analytical solution may prove very difficult to solve as the problem grows in complexity. In the case of modeling eletromigration effects in interconnects, FDM offers us the scalability to handle varying, and often vast, structures.

FDM is a discretization method for finding numerical solutions of dependent variables in a PDE. The local Taylor Expansion of the PDE is used to accomplish this discretization. This method of discretization produces a number of equations which must then be solved for the value of the dependent variable. The PDE can be discretized using many different methods; in our implementation a central difference method is used to discretize the spatial variable x in the electromigration PDE while a first order backward method is used to discretize time t.

$$\frac{\sigma_i^{n+1} - \sigma_i^n}{\Delta t} = \kappa\frac{\sigma_{i+1}^{n+1} - 2\sigma_i^{n+1} + \sigma_{i-1}^{n+1}}{\Delta x^2} \quad (3)$$

In (3), the superscript n indicates the time step and subscript i is the space index, x.

$$-S\sigma_{i+1}^{n+1} + (1-2S)\sigma_i^{n+1} - S\sigma_{i-1}^{n+1} = \sigma_i^n \tag{4}$$

where $S = \kappa\frac{\Delta t}{\Delta x^2}$. This discretization method gives us an implicit scheme for solving the PDE numerically. This means that each discrete point contains data from adjacent time steps which results in a system of equations that must be solved, as apposed to equations that can be solved independently of each other. Implicit schemes allow us more freedom on how large the time step Δt can be and improves stability at the cost of complexity [11].

For the boundary conditions, stress is dependent on the derivative of the stress function. Thus, Neumann boundaries are used and discretized [12].

$$\sigma_x(0,t) = G = \frac{\partial \sigma_i^n}{\partial x} \tag{5}$$

$$\sigma_x(0,t) = \frac{\partial \sigma_i^n}{\partial x} = \frac{\sigma_i^{n+1} - \sigma_{i-1}^{n+1}}{\Delta x} = G \tag{6}$$

We can now use this discretization scheme and plug it into (4) to obtain the following.

$$(S+1)\sigma_i^{n+1} - S\sigma_{i+1}^{n+1} = \sigma_i^n - SG\Delta x \tag{7}$$

Note that we omit the derivation for the second boundary condition $\sigma_x(L,t)$. The resulting system of equations can be mapped to an equation in the form of $A\sigma^{n+1} = \sigma^n$ as seen in (8),

$$\begin{bmatrix} (S+1) & -S & & & & \\ -S & (1-2S) & -S & & & \\ & -S & (1-2S) & -S & & \\ & \ddots & \ddots & \ddots & & \\ & & -S & (1-2S) & -S & \\ & & & -S & (1-2S) & -S \\ & & & & -S & (S+1) \end{bmatrix} \times \begin{bmatrix} \sigma_{left}^{n+1} \\ \sigma_2^{n+1} \\ \vdots \\ \sigma_{x_i-1}^{n+1} \\ \sigma_{right}^{n+1} \end{bmatrix} = \begin{bmatrix} \sigma_{left}^{n} + \beta \\ \sigma_2^{n} \\ \vdots \\ \sigma_{x_i-1}^{n} \\ \sigma_{right}^{n} - \beta \end{bmatrix} \tag{8}$$

where A is a coefficient matrix close to tri-diagonal (in case of one dimension), σ^{n+1} is the vector of unknown stress variables, and σ^n is a vector of already known or previously solved for stress variables. The A matrices were obtained from many specific parameters such as wire lengths, current densities, spatial step sizes and boundary conditions.

The term β is from the boundary condition and is equal to $SG\Delta x$. Each solution of the system of equations results in a vector containing the stress of the wire at a single time step. Subsequent solution of this system, using previous time step solution as the σ^n vector, can produce a vector containing the stress at the respective time steps. By iteratively solving (8), we obtain the transient stress evolution at any places of the wire.

We remark for the sake of presentation, we only consider the one dimensional problem e.g., single or multiple connected wires in one dimension. However, our approach can be easily extended to 2 or 3 dimensions for more complicated structures.

TABLE I

NOTATIONS AND TYPICAL VALUE IN OUR TRANSIENT SIMULATION

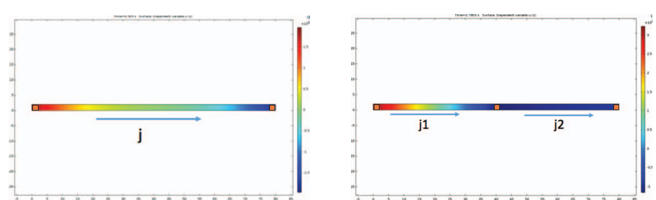
Term	Typical value	Description
ρ	3.00e-8Ω· m	Electrical resistivity
e	1.60e-19C	Electric charge
Z^*	10	Effective valence charge
Ω	1.18e-29 m^3	Atomic volume
k	1.38e-23J/K	Boltzmann constant
B	1.10e11Pa	Back flow stress modular
D_0	7.56e-5m^2/ s	Self-diffusion coefficient
E_a	1.76e-19J	Activation energy
σ_{stress}	500MPa	Critical stress
T	373K	Absolute temperature
δ	3e-7m	Effective thickness of the void interface

The void growth phase follows the void nucleation phase. For void growth phase, the A matrices are similar (with different boundary and initial conditions). The analyzer can take an arbitrary vector of initial conditions or the output stress distribution from the previous void nucleation phase can be used. Once this has been determined by the user, the growth phase portion of the framework operates the same as the void nucleation phase. That is, the new growth phase discretized equation $A\sigma^{n+1} = \sigma^n$ is formed and is then solved iteratively to generate transient data for the wire stress.

IV. NUMERICAL RESULT AND DISCUSSIONS

The proposed FDM-based EM analyzer was prototyped in MATLAB. COMSOL multiphysics [13] is used to validate our proposed analyzer.

In order to validate our result, a FEA tool, COMSOL [13] is used. In the nucleation phase, the initial conditions are set to be zero and default zero flux boundary conditions are used. In the growth phase, initial conditions come from the time at which the cathode in the nucleation phase reaches critical stress. We summarize the major notations and typical parameter values in Table I.



(a) (b)

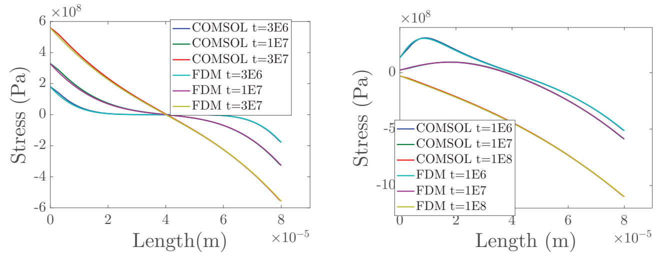
Fig. 3. The testing interconnect structures: (a) 1-wire (b) 2 connected wires

A. *Validation for single 2-terminal wire*

For the 2-terminal single wire as shown in Fig 3(a), tensile stress was generated in the cathode (left node) while the anode node experiences compressive stress. This agrees with our expectations which say that a void should be nucleated at the cathode end of the single wire. Root Mean Squared Error(RMSE) when compared to COMSOL for the void nucleation phase, seen in Fig 4(a), and void growth phase, seen in Fig 4(b), are 0.8033% and 0.4435% respectively.

B. *Validation for straight-line 3-terminal wires*

In the straight-line 3-terminal wires shown in Fig 3(b), current was applied according to the passive/active sink, and

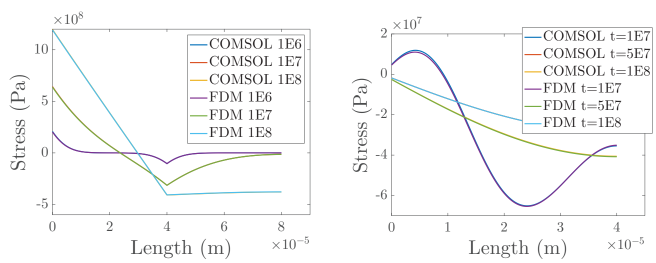


(a) (b)

Fig. 4. EM-stress distribution change over time with $j = 5 \times 10^9 A/m^2$ in single 2-terminal wire for (a) void nucleation and (b) void growth

active reservoir configurations described in [14]. Passive and active refer to the presence of current density in the wire segment where the reservoir segment contains the tensile stress. While it is often the case that voids nucleate in the reservoir segment when the sink is passive, voids can also nucleate in the sink when active. These scenarios are evident in the resulting data which shows both sink and reservoir segments experiencing tensile stress which can result in void nucleation.

While the results agree with our expectations regarding void nucleation and growth in the different wire segments, the agreement of these results with COMSOL data varies between cases. The passive sink case seen in Fig 5 produces accurate results compared to COMSOL data with an average RMSE of 0.203%. However; the active sink Fig 6 and active reservoir Fig 7 still produce good results, but show more variation from the COMSOL data at the boundaries with average RMSEs of 2.1236% and 2.3087% respectively. Similarly, growth phase results for the 2-wire case are worst when the sink is not passive with average RMSEs of 0.4672% for passive sink Fig 5(b), 6.2044% for active reservoir Fig 7(b), and 5.2636% for active sink Fig 6(b).

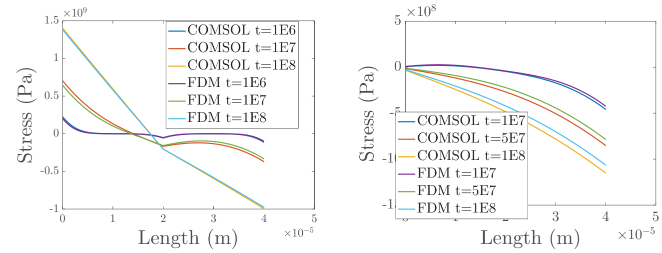


(a) (b)

Fig. 5. 2-wire passive sink case ($j1 = 10^{10} A/m^2$ $j2 = 0$) for (a) void nucleation and (b) void growth

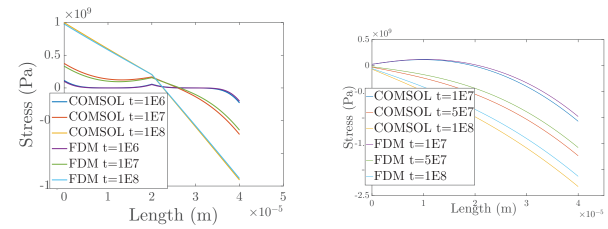
V. Conclusion

In this paper, we proposed an accurate EM analysis model by performing the FDM for EM effects in multi-branch interconnects based on the kinetics of the first principle of EM physics. We started with the fundamental hydrostatic stress evolution PDE for both void growth and nucleation phases with proper boundary and initial conditions for typical multi-branch wires: the single 2-terminal wire, and the straight 3-terminal wires. The new FDM EM analysis easily accommodated existing nonuniform residual stress distribution which is not considered in existing EM models. It considered transient



(a) (b)

Fig. 6. 2-wire active sink case ($j1 = 10^{10} A/m^2$ $j2 = 5 \times 10^9 A/m^2$) for (a) void nucleation and (b) void growth



(a) (b)

Fig. 7. 2-wire active reservoir and active sink case ($j1 = 5 \times 10^9 A/m^2$ $j2 = 10^{10} A/m^2$) for (a) void nucleation and (b) void growth

thermal and current inputs, both of which are difficult for existing models. Numerical results showed that the proposed method agrees with the COMSOL based finite element method in terms of accuracy.

References

[1] J. R. Black, "Electromigration-A Brief Survey and Some Recent Results," *IEEE Trans. on Electron Devices*, vol. 16, no. 4, pp. 338–347, 1969.
[2] I. A. Blech, "Electromigration in Thin Aluminum Films on Titanium Nitride," *Journal of Applied Physics*, vol. 47, no. 4, pp. 1203–1208, 1976.
[3] B. Bailey, "Thermally Challenged," in *Semiconductor Engineering*, 2013.
[4] S. Chatterjee, M. B. Fawaz, and F. N. Najm, "Redundancy-Aware Electromigration Checking for Mesh Power Grids," in *Proc. Int. Conf. on Computer Aided Design (ICCAD)*, pp. 540–547, 2013.
[5] V. Mishra and S. S. Sapatnekar, "The Impact of Electromigration in Copper Interconnects on Power Grid Integrity," in *Proc. Design Automation Conf. (DAC)*, pp. 1–6, 2013.
[6] X. Huang, T. Yu, V. Sukharev, and S. X.-D. Tan, "Physics-based electromigration assessment for power grid networks," in *Proc. Design Automation Conf. (DAC)*, June 2014.
[7] V. Sukharev, "Beyond Black's Equation: Full-Chip EM/SM Assessment in 3D IC Stack," *Microelectronic Engineering*, vol. 120, pp. 99–105, 2014.
[8] H. Chen, S. X.-D. Tan, V. Sukharev, X. Huang, and T. Kim, "Interconnect reliability modeling and analysis for multi-branch interconnect trees," in *Proc. Design Automation Conf. (DAC)*, June 2015.
[9] M. A. Korhonen, P. Borgesen, K. N. Tu, and C. Y. Li, "Stress Evolution Due to Electromigration in Confined Metal Lines," *Journal of Applied Physics*, vol. 73, no. 8, pp. 3790–3799, 1993.
[10] V. Sukharev, A. Kteyan, and X. Huang, "Post-Voiding Stress Evolution in Confined Metal Lines," *IEEE Trans. on Device and Materials Reliability*, vol. 16, no. 1, pp. 50–60, 2016.
[11] N. Ozisik, *Finite Difference Methods in Heat Transfer*. CRC Press, Apr. 1994.
[12] R. LeVeque, *Finite Difference Methods for Ordinary and Partial Differential Equations*. Society for Industrial and Applied Mathematics, 2007.
[13] "Comsol multiphysics." http://www.comsol.com.
[14] M. Lin and A. Oates, "An electromigration failure distribution model for short-length conductors incorporating passive sinks/reservoirs," *IEEE Transactions on Device and Materials Reliability*, vol. 13, pp. 322–326, March 2013.