Passive Modeling of Interconnects By Waveform Shaping *

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ABSTRACT

In this paper, we propose a new approach to enforcing the passivity of a reduced system of general passive linear time invariant circuits. Instead of making the reduced models passive for infinite frequencies, which is difficult and inefficient using state-of-the-art optimization based methods for circuits with many terminals and operating in wideband frequency ranges, the new method works on the signal waveform driving reduced models. It slightly shapes the waveforms of the signal such that the resulting signal spectra are band limited to the frequency range in which the reduced system is passive. As a result, the reduced models only need to be band-limited passive (also called conditionally passive), which can be achieved much easier than traditional passivity for a reduced system, especially for ones with many terminals or requiring wide band accuracy (more poles). We propose to use spectrum truncation via FFT/IFFT and low-pass filter based approaches for transient waveform shaping processing. We analyze the delay and distortion effects caused by using low-pass filters and propose methods to mitigate the two effects. Experimental results on several interconnect circuits demonstrate the effectiveness of the proposed methods.

1. INTRODUCTION

Compact modeling of passive interconnect circuits has been intensively studied in the past due to the urgent need to reduce the increasing circuit complexity as technology scales. Many model order reduction (MOR) techniques have been proposed in the past. The most efficient and successful one is based on subspace projection [14, 3, 15, 11, 4], which was pioneered by Asymptotic Waveform Evaluation (AWE) algorithm [14] where explicit moment matching was used to compute dominant poles at low frequencies. Later more numerical stable techniques are proposed [3, 15, 11, 4] by using implicit moment matching and congruence transform to produce passive models.

Another important development in linear MOR is by means of truncated balanced realization (TBR) methods where the weak uncontrollable and unobservable state variables are truncated to achieve the reduced models [9, 12, 13, 16]. TBR methods can produce nearly optimal models but they are more computationally expensive than projection based methods. Also TBR can produce the passive models in its latest methods [12, 13, 16], which can be approximately viewed as generalized projection-based reduction methods [13].

Although projection based MOR methods are very successful, their applications are mainly limited to RLC circuits. Many high-speed circuits, like RF surface acoustic wave (SAW) filters, spiral inductors, high-speed transmission lines, are still modeled by using measured data (like Scattering parameters) due to many high frequency effects and the freqency dependency of circuit parameters. Another issue with projection based MOR methods is that they become very inefficient for reducing circuits with many terminals in terms of both computational costs and reduced model sizes [6]. The main reason for lost efficiency lies in the fact that with more terminals, more transfer functions are needed to compute and more poles will be used for each increased order of block moments, which is not necessary as polese are system information and should not depend on the terminals.

For generating general-purpose compact models from many measured and simulated data, fitting methods based on least square rational approximation in frequency domain are still widely used [5]. One critical issue in such a modeling process is to preserve the passivity of the original system in the reduced models. Existing approaches like PRIME [10] enforce the passivity by physically realizing each pole/residue (conjugate pole pair) term in the fractional form using Foster's synthesis method. If a pole/residue term can't be realized, it is discarded. As a result, PRIME can lead to very large errors. The latest approach to this general passivity enforcement problem is based on the convex programming (CP) approach by using the state-space representation of the system [2]. The passivity is enforced by using semi-definite constraints during a semi-definite (convex) optimization. But the CP based method suffers very high computational costs and can optimize circuits with less than about 20 poles and 20 terminals in a typical computation setting (on latest Intel Pentium 4 CPU with 1GB memory).

In this paper, we propose a new passivity enforcement approach for general purpose modeling of passive linear circuits. Our new method is based on the observation that most of interconnect circuits like clock trees, substrate, packing, RF passives, and transmission lines are lossy and their frequency responses behave like a band-pass or low-pass filter in general. As a result, the models for those passive systems need not to be passive for all frequencies, as required by traditional passivity enforcement methods. Practically they need only to be passive for a limited bandwidth in which most of the signal energy is concentrated.

Instead of making the reduced models passive for all frequencies, the new method works on the signals going into the reduced models to enforce the passivity. The idea is to slightly shape the waveforms of the signals such that the resulting spectra are bandlimited to the frequency range in which the reduced system is passive. As a result, the reduced models only need to be band-limited passive, which we call conditionally passive in this paper and can be achieved much easier than traditional passivity for a reduced system. We propose two approaches to band-limit (shape) the waveforms. The first method is based on frequency domain fast Fourier transform (FFT) and inverse FFT to explicitly shape the waveforms. The second method is based on insertion of passive low-pass filters (LPF) into the reduced models to implicitly shape the waveforms. For the second method, we analyze the delay and distortion effects introduced by using low-pass filters and propose methods to mitigate the delay effects. Experimental results on several interconnect circuits demonstrate the effectiveness of the proposed methods.

The rest of this paper is organized like this: Section 2 reviews

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the strict (traditional) passivity concepts and their mathematic representation. Section 3 introduces our conditional passivity concept, which will be illustrated by a real circuit example. Section 4 presents our new waveform shaping based approach to the passivity enforcement. Experimental results are given in Section 5. Finally Section 6 concludes this paper.

2. PASSIVITY AND POSITIVE-REALNESS

In this section, we review the standard passivity definition for a network function. Passivity is an important property of many physical systems. A passive network does not generate energy. If the reduced order model (ROM) loses its passivity, it may lead to unbounded responses in transient simulation, which means new energy has been generated in this network.

Fig. 1 shows a transient simulation result of a non-passive circuit under a sinusoidal excitation.

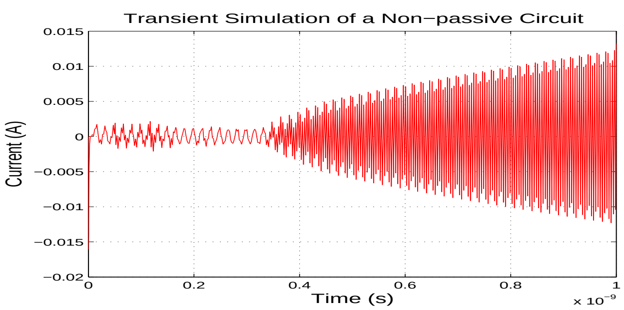

Figure 1: Transient response of a non-passive circuit.

O. Brune [1] has proved that the admittance and impedance matrix of an electrical circuit consisting of an interconnection of a finite number of positive R, positive C, positive L, and transformers are passive if and only if their rational functions are positive real. A network with admittance matrix function $\mathbf{Y}(s)$ is said to be positive real iff

(1) $\mathbf{Y}(s)$ is analytic, for $Re(s) > 0$
(2) $\overline{\mathbf{Y}(s)} = \mathbf{Y}(\bar{s})$, for $Re(s) > 0$
(3) $\mathbf{Y}(s) + \mathbf{Y}(s)^H \geq 0$, for $Re(s) > 0$

Condition (1) means that there are no unstable poles (poles lying on right-half-plane (RHP) in s-domain). Condition (2) refers to system that has real response. And condition (3) is equivalent to the real part of $\mathbf{Y}(s)$ having a positive semi-definite matrix at all frequencies. In other words, the real parts of all the eigenvalues of the $H(s)$ must be equal to or larger than zero. But condition (3) is difficult to satisfy as it requires the checking of frequency responses from DC to infinity.

3. CONDITIONAL PASSIVITY AND CONDITIONAL POSITIVE-REALNESS

In this section, we analyze the relationship of a system's transient responses and its input signals in terms of passivity. We show that a non-passive system can still behave like a passive system when its input signals are band-limited. Such systems can be defined as the conditionally passive and its network functions are conditionally positive-real.

In the previous section, we know for a passive system, its admittance matrix $Y(s)$ needs to be positive real (Re$\{Y(s)\}$ is positive definite) for all frequencies. However, when the $Y(s)$ is not positive real for some frequency ranges, will the system always exhibit non-passive behavior in the time domain as shown in Fig. 1? Actually the answer depends on the spectrum (energy) of the input signal. If the input signal is band-limited to the frequency range where the reduced model is positive real, then the system will still behave passively as the original system.

This can be illustrated by the following example. Fig. 2 shows the frequency responses of a RLC circuit and its reduced model. The two circuits match well below 15Ghz. Above 60Ghz, the real part of the transfer function of the reduced model becomes negative as shown in Fig. 2(a), which means the system becomes non passive. When a sinusoidal input signal of 10Ghz is applied to both systems, we get the exact responses in the time domain as shown in Fig. 3. However, if we apply a sinusoidal signal of 60Ghz, the time-domain responses of the original system and the reduced system will be dramatically different as shown in Fig. 4. The response of the reduced system actually explodes.

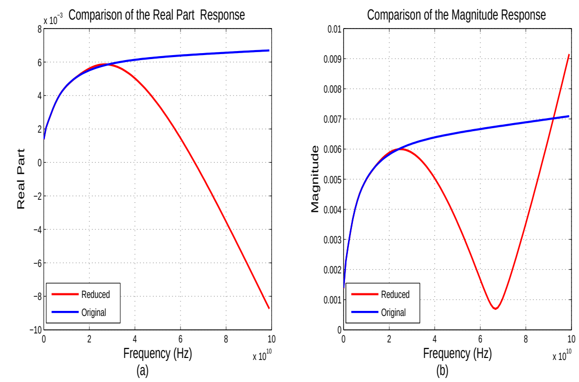

Figure 2: Frequency responses of a reduced model and its original RC circuit.

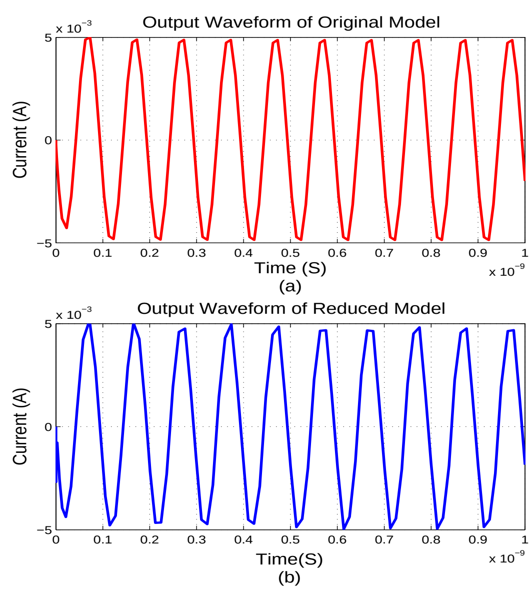

Figure 3: Transient responses of a reduced model and its original RC circuit.

We note that many ideal input signals like Dirac delta function $\delta(t)$, unit step function $u(t) = 1, t \geq 0, u(t) = 0, t < 0$, and unit ramp function $f(t) = t, t \geq 0, f(t) = 0, t < 0$, have an infinite spectrum of frequencies. For example, for Dirac delta function, $L(\delta(t)) = 1$, where $L(X)$ means taking the Laplace transform of function X. So $\delta(t)$ has a constant spectrum for all frequencies and it can easily make any non-passive system to exhibit the non-passive behavior as shown in [7]. The Laplace transform of unit step function and unit ramp function is $1/s$ and $1/s^2$ respectively. When those signals

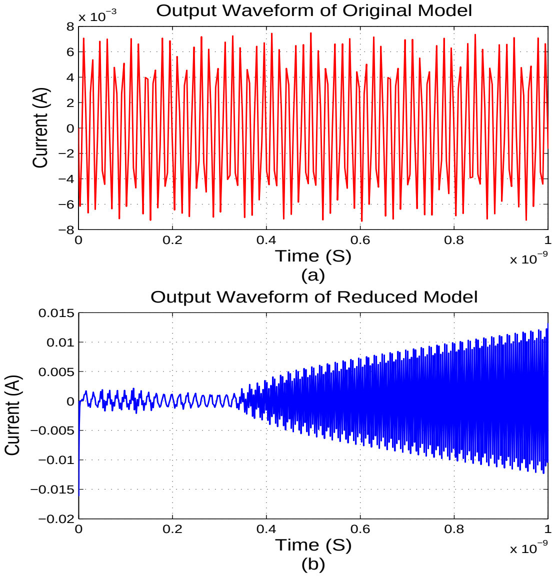

Figure 4: Transient responses of a reduced model and its original RC circuit.

are applied to a non-passive system, non-passive behavior can be easily observed as shown in [11].

However, such ideal signals do not exist in the real world. Most of the active transistors, passive interconnects, RF passive components, and transmission lines exhibit limited bandwidth due to unavoidable capacitive loss, which implies that signal generated by and propagated through those systems will bear limited bandwidth. This situation will become worse as we move to the deep sub 100nm technology. So for realistic signals, we can build a reduced system which is only passive for the given frequency range and the resulting system will still be passive as far as the simulation is concerned. For this purpose, we introduce the conditional passivity and conditional positive-realness.

A network with admittance matrix function $\mathbf{Y}(s)$ is said to be conditionally positive real iff

(cpr1) $\mathbf{Y}(s)$ is analytic, for $Re(s) > 0$
(cpr2) $\overline{\mathbf{Y}(s)} = \mathbf{Y}(\bar{s})$, for $Re(s) > 0 \qquad 0 \leq Im(s) \leq 2\pi f_{max}$
(cpr3) $\mathbf{Y}(s) + \mathbf{Y}(s)^H \geq 0$, for $Re(s) > 0$

In other words, $\mathbf{Y}(s)$ will be positive real for the given frequency range $[0, f_{max}]$.

The main benefit for a reduced system to be conditionally passive is that conditional passivity can be much easier to achieve than strict passivity. Many existing frequency domain rational fitting methods [5, 8] can be used to do this with much more scalable computational costs than the convex programming method [2]. On the other hand, we put more constraints on the signals driving the conditionally passive systems: we need to make sure that the signal spectrum is band limited such that its bandwidth is within the positive real bandwidth of the reduced system. In the following section, we present two methods to achieve this requirement.

4. PASSIVITY ENFORCEMENT BY WAVEFORM SHAPING

In this section, we discuss two methods to band-limit a signal by slightly shaping its waveform. Note that based on the Fourier transform, if a signal is finite in time, its spectrum extends to infinity frequency, and if its bandwidth is finite, its duration is infinite in time. For a practical non-periodic time-limited signal like switching currents in the signal lines due to transistor switching, one can never band limit such a signal from a strictly mathematical point of view. But practically we can make the out-of-band frequency energy sufficiently small compared to the in-band frequency energy such that the out-of-band energy will not stimulate the non-passive behavior of the system.

4.1 FFT and IFFT based Waveform Shaping

The first method is based on the fast Fourier transform (FFT) and inverse fast Fourier transform (IFFT). The idea is to first transform the original transient signal into the frequency domain. Since in FFT (or discrete Fourier transform, DFT), we treat the non-periodic signal as a periodic signal, the resulting signal's spectrum becomes discrete. Then we truncate those frequencies beyond f_{max}, which is given. After this, we perform the inverse FFT on the truncated spectrum to get the time domain waveform of the shaped signal (we only take the waveform in one period). The whole process is illustrated in Fig. 5 and the algorithm is outlined in Fig. 6.

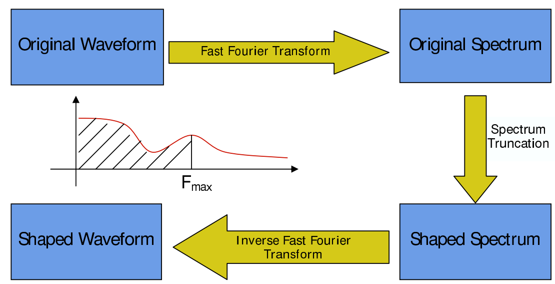

Figure 5: The algorithm flow of FFT and IFFT based waveform shaping.

```
FFT_IFFT_WAVEFORMSHAPING( ) {
Sample input data with F_s;
Fast Fourier Transform
   X_0(k) = Σ_{j=1}^{N} x_0(j) w_N^{(j−1)(k−1)};
Spectrum Truncation
   If f_k < f_max or f_k > F_s − f_max,
    X_1(k) = X_0(k);
   If f_max < f_k < F_s − f_max,
    X_1(k) = 0;
Inverse Fast Fourier Transform
   x_1(j) = (1/N) Σ_{k=1}^{N} X_1(k) w_N^{−(j−1)(k−1)};
return vector: x_1 of length N;
}
```

Figure 6: The algorithm of FFT and IFFT based waveform shaping.

Fig. 7(a)and Fig. 7(b) show a ramp signal and its spectrum. The shaped waveform with the cut-off frequency $f_{max} = 10Ghz$ and the corresponding truncated spectrum are shown in Fig. 7(c) and Fig. 7(d). The shaped waveform with the cut-off frequency $f_{max} = 2Ghz$ and the corresponding truncated spectrum are shown in Fig. 7(e) and Fig. 7(f). In general, the spectrum truncation does not change significantly the waveform characteristics like delay and slew etc. As we truncate high frequency components, the shaped waveform shows some undershoots and overshoots in Fig. 7(e). Those small undershoots and overshoots do not affect delay and timing of the shaped waveform when it propagates through the reduced model. If we truncate the spectrum at a higher frequency such as 10GHz, we find that the resulting waveform is almost the same as the original one, which is shown in Fig. 7(c). This demonstrates that if the cutoff frequency is high enough, the distortion caused by truncating can be tolerated.

The drawback of the explicit waveform shaping method using FFT and IFFT is that it takes extra computational costs to process the signals. The computational costs are $O(nlog_2(n))$, where n is the number of sampling points.

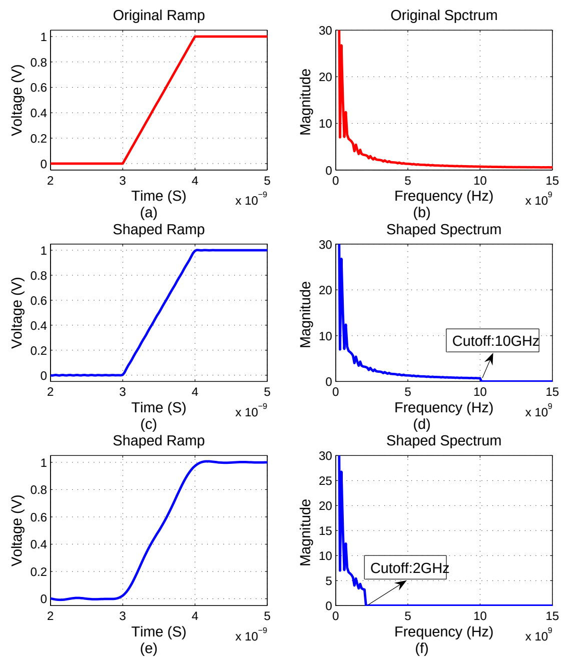

Figure 7: A ramp signal shaped at different frequencies.

4.2 Low-Pass Filter Based Waveform Shaping

The second method is based on the implicit waveform shaping by adding passive low-pass filters between the input signal and the reduced system as shown in Fig. 8. In this way, we guarantee that the signals through the reduced system are band-limited.

Notice if we have a few input terminals (as for many interconnect circuits like clock trees or clock meshes), adding a few filters at those terminals will not increase the sizes of the reduced models significantly.

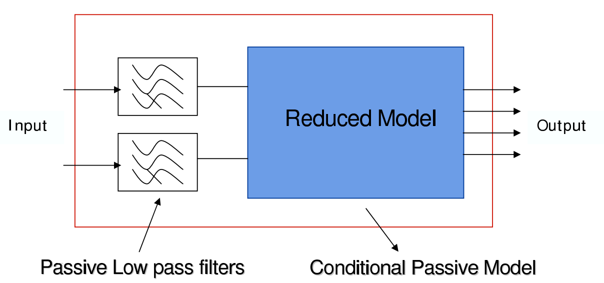

Figure 8: Low pass filter based waveform shaping.

Since the filter can be passively realized by LC ladder, it can be combined with the reduced model to function as a passive model. Therefore, we can conveniently use this new model in current simulation software such as Spice.

However, we need to look at several issues associated with this method before we use it. First, the low-pass filter can also distort the input signals as different frequency components may be delayed differently. Second, the introduction of low-pass filter can introduce delay. In the following, we discuss methods to mitigate those two problems.

4.2.1 Mitigation of Distortion Problems

The phase function and the resulting group delay function of a filter have profound time domain ramifications as they have a direct effect on the waveform shape of the output signals. As a result, we choose Bessel filter family due to its good time domain property. A Bessel filter has a linear phase characteristic over the pass-band of the filter, which implies a constant time delay over the pass-band of the filter (see Fig. 9) so that the phase distortion in the filtering process can be avoided. From Fig. 9(a), we can see a constant time delay from DC to the normalized frequency 1 when the order (n) of filter is higher than 3. In addition, its step response exhibits negligible overshoot and ringing.

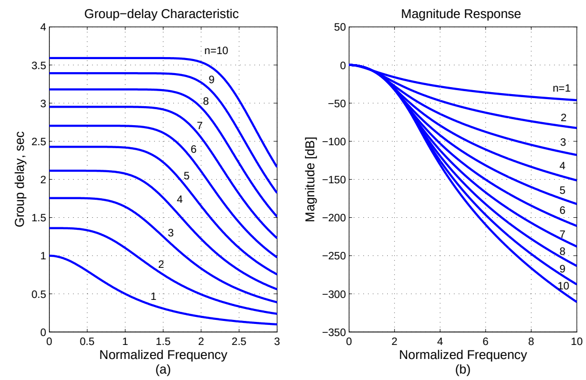

Figure 9: Group-delay characteristic and magnitude response for different order Bessel filters (normalized frequency).

However, a gradual roll-off (longer decay range) is the price we have to pay for a good time domain property. Fortunately, we can compromise it by increasing the order of the filter (see Fig. 9(b)) at the cost of larger reduced models. Another way is to increase the passive frequency range so that the filter has sufficient reduction of spectrum (again at the cost of larger reduced models).

4.2.2 Mitigation of Delay Problems

Another issue we have to take into consideration is the time delay caused by the filter. Three factors can influence the time delay: the prototype, the order, and the cutoff frequency of a filter. Among them, the cutoff frequency is the dominant factor because the delay is inversely proportional to the cutoff frequency of a filter.

$$\text{Actual Delay} = \frac{\text{Normalized Delay}}{\text{Actual Corner Frequency (fc)}} \tag{1}$$

For example, for the eighth order Bessel filter, the normalized delay is 2.703s. If the cutoff frequency is as high as 20GHz, the actual delay could be as small as 0.135ns.

Hence, if the cutoff frequency is sufficiently high, the group delay caused by the filter can be made sufficiently small compared to the delay of the original circuit so that such a delay can be ignored.

5. EXPERIMENTAL RESULTS

In this section, we present some experimental results on two interconnect circuits from our industry partner. All the experimental results are conducted on a computer with AMD Athlon(64) 3800+ 2.41Ghz CPU and 500MB DDR memory. The conditional passivity is achieved by using the minimum square fitting method on the required transfer functions with poles computed from projection based methods like PRIMA. This fitting method can make the reduced models accurate to the given maximum frequency and ensure the passivity of the models in the given frequency range.

The first example is a RC circuit with 210 nodes and 3 terminals. In this experiment, we use a steep square waveform as the input signal, as shown in Fig. 10(a). We apply this signal to the original model, the reduced model, and the LPF (low-pass filter) based reduced model. The output waveforms of these three models are shown in Fig. 10(b)(c)(d), respectively.

The reduced model is conditionally passive: the passivity of the model can only be preserved at the frequency range from DC to 15GHz. Since a steep square waveform contains high frequency components beyond this range, we can observe the erratic time-domain behavior caused by energy generated at high frequencies, as shown in Fig. 10(c).

However, by eliminating those high frequency components by LPF, the output waveform of the LPF based reduced model (Fig. 10(d)) matches the output waveform of the original model (Fig. 10(b)) with little discrepancy. Therefore, the LPF based reduced model can function as a passive model at all frequencies.

In addition, we compare the qualities of Bessel LPF based reduced model and Ellipse LPF based reduced model in Fig. 11. The Fig. 11(a) and Fig. 11(b) show the transient responses from the original circuit due to the square input waveform. Fig. 11(c) and Fig. 11(d) show the transient responses from Bessel LPF based reduced model while Fig. 11(e) and Fig. 11(f) are the transient responses from Ellipse based reduced models using the same filter order. So the results clearly show that the Bessel LPF reduced model is superior to the Ellipse based models. As shown in (Fig. 11(d)(f)), Bessel LPF based reduced model can effectively avoid the overshoots and ringings. This result further demonstrates the rational of our choice for Bessel LPF over other types of LPFs.

The second example is a RC circuit (168 nodes) with 132 terminals (14 drivers and 118 receivers). This circuit does not have much to reduce due to large number of terminals compared to the number of nodes. But it serve as an example that the convex programming method fails to optimize due to the large terminal count. Still we use fitting method to do the frequency domain reduction and make the reduced models accurate to 50Ghz. We use a steep square waveform as shown in Fig. 12(a) as the inputs. We apply this signal to the original model, the reduced model, and the LPF based reduced model. The output waveforms of these three models are shown in Fig. 12(b)(c)(d), respectively.

By eliminating high frequency components by LPF. The results are similar: output waveforms from the LPF based reduced model (Fig. 12(d)) match well the original model (Fig. 12(b)). But the simple reduced models lead to erratic time-domain behavior due to its non-passivity at high frequencies as shown in Fig. 12(c).

The experimental results also show that the output of LPF based reduced model exhibits less ringing than the output of original model. This is because the ringing is caused by high frequency components of input signal and many of those components are eliminated by LPF in the reduced model. If the those ringings are of interests, we can observe more of them by increasing the frequency range of the reduced models.

6. CONCLUSION

In this paper, we have proposed a new passivity enforcement scheme for passive modeling of general linear time-invariant systems. Instead of making the model passive for all frequencies as traditional methods did, the new method works on the signal waveforms assuming the circuit models are only passive for limited frequency band (called conditionally passive). Such relaxation makes the circuit passive modeling work much easy using fitting based methods for a reduced system especially for systems with many terminals or requiring wide band accuracy using measured data. By shaping the signal waveforms such that resulting signal's spectra are band-limited, the resulting systems will still be passive from simulation's perspective.

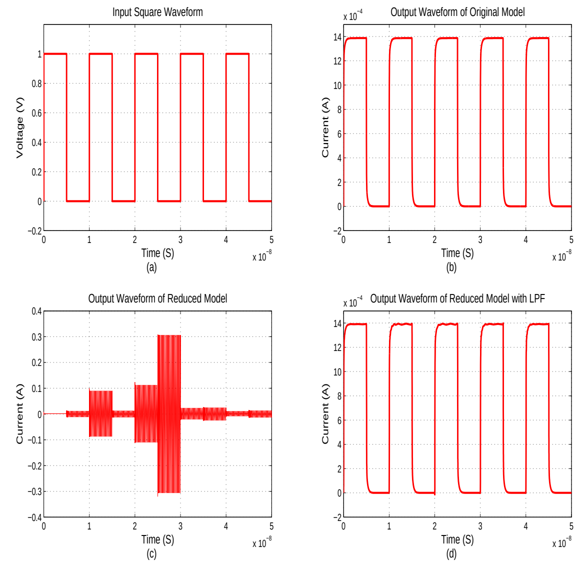

Figure 10: The comparison of responses of different models in time domain for the first example.

We proposed to use frequency truncation via FFT/IFFT and low-pass filter based approaches for transient waveform shaping processes. We analyzed the delay and distortion effects caused by using low-pass filters and proposed methods to mitigate the distortion and delay effects. Experimental results on several interconnect circuits demonstrated the effectiveness of the proposed method.

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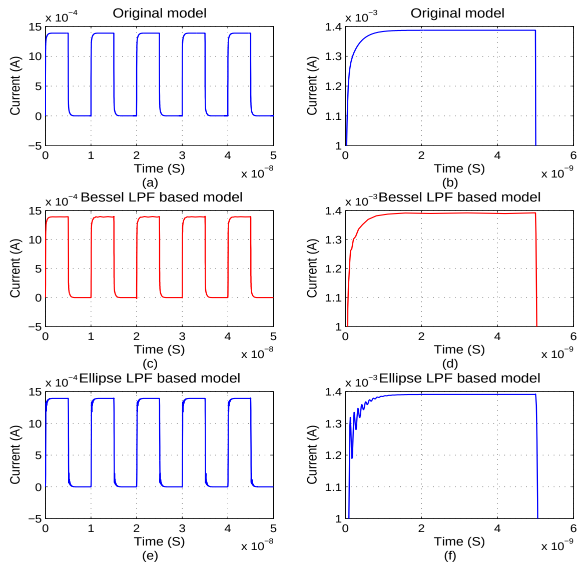

Figure 11: Comparison in time domian between reduced models based on Bessel filters and Ellipse filters.

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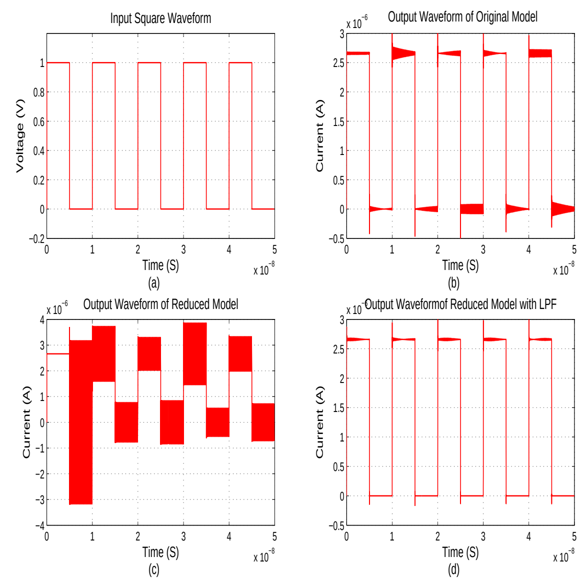

Figure 12: The comparison of responses of different models in time domain for second example.