A Survey of RLCK Reduction and Simulation Methods by Fast Truncated Balanced Realization *

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ABSTRACT

Model order reduction by truncated balanced realization (TBR) is better than Krylov subspace methods to achieve smaller models with global error control. TBR projects a system onto the dominant invariant subspace in terms of both controllability and observability measured by their gramians. However, to obtain two gramians, two Lyapunov equations have to be solved and its high computation costs involved limit its application to only small circuits. To mitigate this problem, several methods are proposed to obtain the approximated dominant subspace of the gramian (or gramian product). In this paper, we survey several recently proposed fast TBR methods by gramian approximation techniques for the model order reduction and simulation of RLCK circuits. We present the pros and cons of each method and compare them on some large RLCK circuits.

1. INTRODUCTION

Model order reduction (MOR) is an efficient technique to reduce the circuit complexity while producing a good approximation of the input and output behavior. Projection based techniques can be classified into Krylov subspace (moment-matching) based methods and balanced truncation based methods. Krylov subspace based approaches have been a great success in the reducing the interconnect circuits [2, 6, 3, 8, 9, 10]. Latest developments allow implicit moment-matching in a projection framework and stability, passivity and structure information inherent to RLCK circuits can be preserved easily by exploiting the internal structure of RLCK formulation.

While suitable for reduction of large-scale circuits, those techniques do not necessarily generate models as compact as desired. Therefore, another approach, Truncated Balanced Realization (TBR), or balanced truncation, which has been well developed in the control community [5], has been studied intensively in design automation community. However, balanced truncation methods are too expensive to directly apply to integrated circuit problems due to the cubic cost to solve two Lyapunov equations. To remedy this problem, several gramian approximation methods have been proposed [7, 12, 4], where the approximated dominant subspace of a gramain can be obtained in a very efficient way. The single gramian (SGA) approximation technique (called Poor Man's TBR) [7] was first proposed for RC circuit, which can be naturally formulated in a first-order form with matrices both symmetric and positive-definite. In this case, two gramians equal and can be simultaneously diagonalized via a congruency transformation. As a result, both good accuracy and passivity can be preserved simultaneously. However, for general RLCK circuits, the first-order formulation could be either symmetric or positive-definite. Therefore, to preserve better accuracy or passivity cannot be archived as the same time.

There are several methods proposed to mitigate the problem. One of them, SBPOR [11], is based on the second-order formulation, which is both symmetric and positive-definite for general RLCK circuits. In this method, second-order gramians are defined based on a symmetric first-order realization. As a result, both second-order gramians, the leading blocks of the gramians of first-order realization, equal and can be simultaneously diagonalized by a congruency transformation. As a result, a provably passive reduced second-order model can be obtained by projecting the original system onto the approximated dominant invariant subspace of one second-order gramian (SOGA) [12].

Other gramian approximation methods covered in this survey include double gramian approximation (DGA), cross-gramain approximation (CGA) and recently proposed response gramian approximation (RGA) for power grid network analysis. CGA [7] combines both controllability and observability into a single cross-gramian. The latter, ETBR [4], is used to simulate circuits with a large number of independent sources, which makes conventional multi-port model reduction ineffective. ETBR considers both system as well as the input sources for reduction by defining a approximated response gramian.

2. CIRCUITS FORMULATION

Circuit formulations are dynamical systems with special internal structures. In this paper, two kinds of formulations, NA and VNA, are used, both of which stamp L^{-1} instead of inductance matrix L because L^{-1} matrix is easier to sparsify.

2.1 Second-order circuit formulation

ENOR [8] stamps the nodal susceptance $\Gamma = E_l L^{-1} E_l{}^T$ (E_l is the incident matrix for inductance) and results in a second-order NA (nodal analysis) form, which can be passively reduced [8, 9]. The second-order formulation is as follows

$$\begin{aligned} &C\dot{v}(t) + Gv(t) + \Gamma \int v(t) = Bi(t) \\ &y(t) = B^T v(t) \end{aligned} \tag{1}$$

where $i(t), y(t) \in R^p$ are input currents and output voltages; $v(t) \in R^n$ are nodal voltages; $G, C, \Gamma \in R^{n\times n}$ are matrices of conductance, capacitance and susceptance respectively; $B \in R^{n\times p}$ is the input matrix and its transpose $B^T \in R^{p\times n}$ is the output matrix. An important property is that, in the second-order formulation, the system matrices are both symmetric and positive semi-definite

$$C = C^T \geq 0 \quad G = G^T \geq 0 \quad \Gamma = \Gamma^T \geq 0 \tag{2}$$

which means the realization is both symmetric and passive.

2.2 First-order circuit formulation

Recently, the branch vector potential $A_l = \int vE_l$ is introduced as a new state variable to obtain a first-order admittance that contains L^{-1} elements [13]. If we define $x^T = [v^T, A_l^T]$, the first-order formulation, called vector nodal analysis (VNA), is as follows

$$\begin{aligned} &\mathcal{C}\dot{x}(t) = -\mathcal{G}x(t) + \mathcal{B}i(t) \\ &y(t) = \mathcal{B}^T x(t) \end{aligned} \tag{3}$$

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where

$$\mathcal{C}=\begin{bmatrix} C & 0 \\ 0 & L^{-1} \end{bmatrix}, \mathcal{G}=\begin{bmatrix} G & E_l L^{-1} \\ -L^{-1}E_l{}^T & 0 \end{bmatrix}, \mathcal{B}=\begin{bmatrix} B \\ 0 \end{bmatrix} \quad (4)$$

Like MNA formulation, VNA matrices have the following properties:

$$\mathcal{C}=\mathcal{C}^T \geq 0 \quad \mathcal{G}+\mathcal{G}^T \geq 0 \quad (5)$$

Hence the the formulation is a passive form and it can lead to passive reduction. Note that, such a formulation is NOT symmetric because $\mathcal{G}$ is not symmetric. It is easy to verify both formulations (1) (4) have the same transfer function

$$H(s)=B^T(Cs+G+\Gamma/s)^{-1}B \quad (6)$$

as such they are equivalent in terms of input-output behavior and (4) can be viewed as a first-order realization of (1).

3. TRADITIONAL REDUCTION METHODS

3.1 Projection-based MOR

Given a first-order system in descriptor form

$$\begin{aligned} \mathcal{E}\dot{x}(t) &= \mathcal{A}x(t)+\mathcal{B}u(t) \\ y(t) &= \mathcal{C}x(t) \end{aligned} \quad (7)$$

where $\mathcal{E}\in R^{n\times n}$, $\mathcal{A}\in R^{n\times n}$, $\mathcal{B}\in R^{n\times p}$, $\mathcal{C}\in R^{p\times n}$, $y(t)$, $u(t)\in R^p$, typically, we have $p\ll n$. Model reduction algorithms seek to produce a smaller system

$$\begin{aligned} \tilde{\mathcal{E}}\dot{x}(t) &= \tilde{\mathcal{A}}x(t)+\tilde{\mathcal{B}}u(t) \\ y(t) &= \tilde{\mathcal{C}}x(t) \end{aligned} \quad (8)$$

where $\tilde{\mathcal{E}}\in R^{r\times r}$, $\tilde{\mathcal{A}}\in R^{r\times r}$, $\tilde{\mathcal{B}}\in R^{r\times p}$, $\tilde{\mathcal{C}}\in R^{p\times r}$, $y(t)$, $u(t)\in R^p$. Order r is much smaller than the original order n, i.e. $r\ll n$, but the output $y(t)$ and $\tilde{y}(t)$ are approximately equal for inputs $u(t)$ of interest. This can be achieved by constructing matrices $\mathcal{W}$ and $\mathcal{V}$ whose columns span a useful subspace, and projecting the original equations in the column spaces of $\mathcal{W}$ and $\mathcal{V}$

$$\tilde{\mathcal{E}}=\mathcal{W}^T\mathcal{E}\mathcal{V}, \tilde{\mathcal{A}}=\mathcal{W}^T\mathcal{A}\mathcal{V}, \tilde{\mathcal{B}}=\mathcal{W}^T\mathcal{B}, \tilde{\mathcal{C}}=\mathcal{C}\mathcal{V} \quad (9)$$

Often the transfer functions

$$\begin{aligned} \mathcal{H}(s) &= \mathcal{C}(s\mathcal{E}-\mathcal{A})^{-1}\mathcal{B} \\ \tilde{\mathcal{H}}(s) &= \tilde{\mathcal{C}}(s\tilde{\mathcal{E}}-\tilde{\mathcal{A}})^{-1}\tilde{\mathcal{B}} \end{aligned} \quad (10)$$

are used as a metric for approximation: if $\|\mathcal{H}(s)-\tilde{\mathcal{H}}(s)\|\leq\varepsilon$, in some appropriate norm, for some given allowable error ε and allowed domain of the complex frequency variable s, the reduced model is accepted as accurate.

3.2 Classical truncated balanced realization (TBR) methods

Given a first-order system in descriptor form (7), assume $\mathcal{E}$ is nonsingular, controllability and observability gramians are the unique symmetric positive definite solutions to the generalized Lyapunov equations

$$\begin{aligned} \mathcal{A}\mathcal{X}\mathcal{E}^T+\mathcal{E}\mathcal{X}\mathcal{A}^T+\mathcal{B}\mathcal{B}^T=0 \\ \mathcal{A}^T\mathcal{Y}\mathcal{E}+\mathcal{E}^T\mathcal{Y}\mathcal{A}+\mathcal{C}^T\mathcal{C}=0 \end{aligned} \quad (11)$$

Since the eigenvalues of the product $\mathcal{X}\mathcal{Y}$ are invariant under similarity transformation, we can perform a similarity transformation to diagonalize the product $\mathcal{X}\mathcal{Y}$ such that

$$\mathcal{T}^{-1}\mathcal{X}\mathcal{Y}\mathcal{T}=\Sigma=diag(\sigma_1{}^2,\sigma_2{}^2,\ldots,\sigma_n{}^2) \quad (12)$$

where the Hankel singular values of the system (σ_k) are arranged in a descending order. If we partition the matrices as

$$\begin{bmatrix} \mathcal{W}_1^T \\ \mathcal{W}_2^T \end{bmatrix}\mathcal{X}\mathcal{Y}\begin{bmatrix} \mathcal{V}_1 & \mathcal{V}_2 \end{bmatrix}=\begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \quad (13)$$

where $\Sigma_1=diag(\sigma_1{}^2,\sigma_2{}^2,\ldots,\sigma_r{}^2)$ are the first r largest eigenvalues of gramian product XY and V_1 and W_1 are corresponding eigenvectors. A reduced model (8) can be obtained by projection onto the invariant subspaces associated with the dominant eigenvalues of the product of gramians XY

$$\tilde{\mathcal{E}}=\mathcal{W}_1{}^T\mathcal{E}\mathcal{V}_1, \tilde{\mathcal{A}}=\mathcal{W}_1{}^T\mathcal{A}\mathcal{V}_1, \tilde{\mathcal{B}}=\mathcal{W}_1{}^T\mathcal{B}, \tilde{\mathcal{C}}=\mathcal{C}\mathcal{V}_1 \quad (14)$$

The error in the transfer function of the order r approximation is bounded by

$$\|\mathcal{H}(s)-\tilde{\mathcal{H}}(s)\|=2\sum_{i=r+1}^{n}\sigma_k \quad (15)$$

In the classical method, the computational cost is dominated by solving Lyapunov equations $O(n^3)$, which makes it too expensive to apply to integrated circuits problem and thus an efficient gramian approximation technique is highly appreciated.

4. FAST TBR VIA GRAMIAN APPROXIMATION

4.1 Single gramian approximation (SGA)

The first-order circuit formulation (4) falls into the class of system in descriptor form (8). In frequency domain, the controllability gramian $\mathcal{X}$ can also be computed from the expression

$$\mathcal{X}=\frac{1}{2\pi}\int_{-\infty}^{+\infty}(j\omega\mathcal{C}+\mathcal{G})^{-1}\mathcal{B}\mathcal{B}^T(j\omega\mathcal{C}+\mathcal{G})^{-H}dw \quad (16)$$

where superscript H denotes Hermitian transpose. A gramian approximation process was proposed in [7]. Let ω_k be kth sampling point. If we define

$$z_k=(j\omega_k\mathcal{C}+\mathcal{G})^{-1}\mathcal{B} \quad (17)$$

then $\mathcal{X}$ can be approximated as

$$\hat{\mathcal{X}}=\sum z_k z_k^H=\mathcal{Z}\mathcal{Z}^H \quad (18)$$

where $Z=[z_1,z_2,\ldots,z_q]$. If we peforme a singular value decomposition on $Z=\hat{U}\hat{\Sigma}\hat{V}$, then we have

$$\begin{aligned} &\hat{X}=ZZ^H=(\hat{U}\hat{\Sigma}\hat{V})(\hat{U}\hat{\Sigma}\hat{V})^T=\hat{U}\hat{\Sigma}^2\hat{U}^T= \\ &\begin{bmatrix} \hat{U}_1 & \hat{U}_2 \end{bmatrix}\begin{bmatrix} \hat{\Sigma}_1^2 & 0 \\ 0 & \hat{\Sigma}_2^2 \end{bmatrix}\begin{bmatrix} \hat{U}_1^T \\ \hat{U}_2^T \end{bmatrix} \end{aligned} \quad (19)$$

where $\hat{U}^T\hat{U}=I$. $\hat{U}$ converges to the eigenspaces of X and the dominant eigenvectors $\hat{U}_1$ can be used as the projection matrix in a model reduction approach

$$\tilde{\mathcal{C}}=\hat{U}_1^T\mathcal{C}\hat{U}_1, \tilde{\mathcal{G}}=\hat{U}_1^T\mathcal{G}\hat{U}_1, \tilde{\mathcal{B}}=\hat{U}_1^T\mathcal{B} \quad (20)$$

This kind of transformation is known as congruency transformation, which preserves the definiteness of matrices such that $\tilde{\mathcal{C}}=\tilde{\mathcal{C}}^T\geq 0, \tilde{\mathcal{G}}+\tilde{\mathcal{G}}^T\geq 0$, implying the reduced-order system has guaranteed passivity [6]. Given q sampling points and p inputs, the cost of SVD on matrix $Z_{n\times pq}$ is $O(n(pq)^2)$. In addition, it takes q matrix factorizations and pq matrix solves. The total cost is $O(n(pq)^2+qn^{\beta}+pqn^{\alpha})$ (typically, $1.1\leq\beta\leq 1.5$ and $1\leq\alpha\leq 1.2$ for circuits) [7], which is dominated by $O(qn^{\beta})$. However, since the system is projected onto the dominant invariant subspace of controllability gramian only instead of the gramian product, the system is not balanced and the accuracy will be sacrificed.

4.2 Double gramian approximation (DGA)

In fact, if the passivity constraint is removed, controllability and observability gramians can be approximated simultaneously. It is easy to see the formulation(4) can be rewritten into a symmetric formulation with the following system matrices

$$\mathcal{C}=\begin{bmatrix} C & 0 \\ 0 & -L^{-1} \end{bmatrix}, \mathcal{G}=\begin{bmatrix} G & E_l L^{-1} \\ L^{-1}E_l{}^T & 0 \end{bmatrix}, \mathcal{B}=\begin{bmatrix} B \\ 0 \end{bmatrix} \quad (21)$$

Note that, both $\mathcal{G}$ and $\mathcal{C}$ is symmetric but they are NOT positive semi-definite anymore. In this symmetrized case (4), both gramians are equal $\mathcal{Y} = \mathcal{X}$. Since the gramian $\mathcal{X}$ is symmetric, it is orthogonally diagonalizable, i.e., there exists $\mathcal{T}^{-1} = \mathcal{T}^T$ such that $\mathcal{T}^T \mathcal{X} \mathcal{T} = \Sigma$. Then, we have

$$\mathcal{T}^T \mathcal{X}\mathcal{X}\mathcal{T} = (\mathcal{T}^T \mathcal{X}\mathcal{T})(\mathcal{T}^T \mathcal{X}\mathcal{T}) = \Sigma^2 \tag{22}$$

which means, in this symmetric case, the eigenspace of gramian product $\mathcal{X}\mathcal{X}$ is exactly the eigenspace of each gramian $\mathcal{X}$ and we only need to find the dominant invariant subspace of an approximated gramian $\hat{\mathcal{X}}$. So the same gramian approximation process from (16) to (20) can be performed. However, the passivity is not guaranteed as the reduced matrices are no more positive semi-definite.

There is always a tradeoff in the first-order circuit formulation, either symmetric (implying accuracy) or positive semi-definite (implying passivity). Both can be obtained simultaneously only when the circuit is an RC circuit, where $\mathcal{G} = G, \mathcal{C} = C, \mathcal{B} = B$.

4.3 Second-order gramian aproximation (SOGA)

In order to balance accuracy and passivity, a second-order balanced truncation method was proposed [12], where second-order gramians are defined based on a symmetric first-order realization (21). As we know, both gramians are equal in this symmetric case. If we compatibly partition the gramians as

$$\mathcal{X} = \mathcal{Y} = \begin{bmatrix} R & S \\ S^T & F \end{bmatrix} \tag{23}$$

then the second-order gramians, which measures the contribution of the node voltages v with respect to the transfer function [12], are also equal

$$X_2 = Y_2 = R \tag{24}$$

Since the gramians are equal, we only need to approximate the dominant invariant subspace of one second-order gramian R. Given the symmetric realizaton (21), starting from (18), if we compatibly partition $\mathcal{Z}^H = \begin{bmatrix} Z_1^H & Z_2^H \end{bmatrix}$, then we have

$$\hat{\mathcal{X}} = \begin{bmatrix} \hat{R} & \hat{S} \\ \hat{S}^T & \hat{F} \end{bmatrix} = \begin{bmatrix} Z_1 Z_1^H & Z_1 Z_2^H \\ Z_2 Z_1^H & Z_2 Z_2^H \end{bmatrix} \tag{25}$$

So the approximated second-order gramian $\hat{R}$ equals $Z_2 Z_2^H$ and can be diagonalized as follows

$$\begin{aligned} \hat{R} = Z_1 Z_1^H = (\hat{U}\hat{\Sigma}\hat{V})(\hat{U}\hat{\Sigma}\hat{V})^T = \hat{U}\hat{\Sigma}^2\hat{U}^T = \\ \begin{bmatrix} \hat{U}_1 & \hat{U}_2 \end{bmatrix} \begin{bmatrix} \hat{\Sigma}_1^2 & 0 \\ 0 & \hat{\Sigma}_2^2 \end{bmatrix} \begin{bmatrix} \hat{U}_1^T \\ \hat{U}_2^T \end{bmatrix} \end{aligned} \tag{26}$$

A reduced model can be obtained by projecting onto $\hat{U}_1$

$$\tilde{C} = \hat{U}_1^T C \hat{U}_1 \quad \tilde{G} = \hat{U}_1^T G \hat{U}_1 \quad \tilde{\Gamma} = \hat{U}_1^T \Gamma \hat{U}_1 \quad \tilde{B} = \hat{U}_1^T B \tag{27}$$

Since the congruency transformation preserves the symmetry and definiteness of matrices we have

$$\tilde{C} = \tilde{C}^T \geq 0 \quad \tilde{G} = \tilde{G}^T \geq 0 \quad \tilde{\Gamma} = \tilde{\Gamma}^T \geq 0 \tag{28}$$

and the reduced-order system has guaranteed passivity [9].

4.4 Cross-gramian approximation (CGA)

The cross-gramian $\mathcal{X}_{CG}$ is introduced which contains both controllability and observability information in a single matrix. Given a system in descriptor form (7), $\mathcal{X}_{CG}$ can be calculated from the Sylvester equation

$$\mathcal{A}\mathcal{X}_{CG}\mathcal{E} + \mathcal{E}\mathcal{X}_{CG}\mathcal{A} = -\mathcal{B}\mathcal{C} \tag{29}$$

The cross-gramian reduction method is based on projecting onto the eigenspaces related to the dominant eigenvalues of $\mathcal{X}_{CG}$. For the symmetric systems, both of the two Lyapunov equations used in balanced truncation will be the same as the Sylvester equation in the cross-gramian method. Also, both controllability and observability gramians are identical to the cross-gramian and $\mathcal{X}\mathcal{Y} = \mathcal{X}_{CG}^2$. These properties make the cross-gramian method equivalent to the balanced truncation method for symmetric models. Given the circuit formulation (4), in the frequency domain, $\mathcal{X}_{CG}$ is expressed as

$$\mathcal{X}_{CG} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} (j\omega\mathcal{C} + \mathcal{G})^{-1}\mathcal{B}\mathcal{B}^T (j\omega\mathcal{C} + \mathcal{G})^{-1} dw \tag{30}$$

Let ω_k be kth sampling point. If we define

$$z_{c_k} = (j\omega_k\mathcal{C} + \mathcal{G})^{-1}\mathcal{B} \quad z_{o_k} = (j\omega_k\mathcal{C} + \mathcal{G}^T)^{-1}\mathcal{B} \tag{31}$$

then $\hat{X}_{CG}$ can be computed as

$$\hat{X}_{CG} = \sum z_{c_k} z_{o_k}^H = Z_c Z_o^H \tag{32}$$

where Z_c and Z_o is are matrices whose columns are z_{c_k} and z_{o_k} respectively. In order to find the eigenvectors of X_{CG}, we need to do the eigendecomposition on $Z_c Z_o^H$ and sort the eigenvalues

$$\begin{bmatrix} \tilde{W}_1^T \\ \tilde{W}_2^T \end{bmatrix} Z_c Z_o^H \begin{bmatrix} \tilde{V}_1 & \tilde{V}_2 \end{bmatrix} = \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \tag{33}$$

A reduced order model can be obtained by a projection onto the dominant eigenspace $\tilde{W}_1$ and $\tilde{V}_1$. Note that, since the projection is not done by a congruency transformation ($\tilde{W}_1 \neq \tilde{V}_1$), the passivity of the reduced model is not guaranteed. In addition, since an eigenanalysis of a full matrix is still needed, the computational cost can not be reduced too much.

4.5 Response grammian approximation (RGA)

Model order reduction becomes inefficient when the number of ports is large. However, when input information is available a priori, the issue can be resolved by including the input signals as part of the system and converting an MIMO system into an SIMO system. In ETBR (extended TBR) method proposed in [4], a response grammian at the frequency domain is defined as

$$\mathcal{X}_r = \frac{1}{2\pi}\int_{-\infty}^{+\infty} (j\omega\mathcal{C} + \mathcal{G})^{-1}\mathcal{B}u(j\omega)u^T(j\omega)\mathcal{B}^T (j\omega\mathcal{C} + \mathcal{G})^{-H} d\omega \tag{34}$$

Let ω_k be kth sampling point over the frequency range. If we further define

$$z_k = (j\omega_k\mathcal{C} + \mathcal{G})^{-1}\mathcal{B}u(j\omega_k) \tag{35}$$

then $\mathcal{X}_r$ can be approximately computed as:

$$\hat{X}_r = \sum z_k {z_k}^H = Z_r Z_r^H \tag{36}$$

where Z_r is a matrix whose columns are z_k. The projection matrix can be obtained by performing a singular value decomposition on Z_r. After this, we can reduce the original matrices into small ones and then perform the transient analysis on the reduced circuit matrices. Notice that we need frequency response of input signal $u(j\omega_k)$ in (35). This can be obtained by fast Fourier transformation on the input signals in time domain.

5. EXPERIMENTAL RESULTS

In the section, we present some experimental results to compare those methods covered in this paper. The original model is an RLCK circuit with an order of 640. The reduced order is set to be 20 for all methods. As shown in Fig. 1, graiman approximation based methods (SGA, DGA, CGA, SOGA) deliver a better approximation than the Krylov subspace based second order method: SAPOR [9] over a wide frequency band. However, SAPOR is more accurate in low frequency band close to the expansion point ($0.1GHz$).

Among all gramian approximation methods in terms of accuracy, DGA is the best because it takes into consideration both controllability and observability gramians. SOGA is in the second place as

Table 1: Time complexity comparison

Methods	Gramians used	Passivity	CPU cost
SGA	contr.	yes	$O(qn^{\beta})$
DGA	contr. observ.	no	$O(qn^{\beta})$
CGA	cross	no	$O(n^3)$
SOGA	contr. observ (2nd)	yes	$O(qn^{\beta})$

$1.1 \leq \beta \leq 1.5$ for sparse matrices. q is the number of sampling points.

it considers second-order controllability and observability gramians, which are leading blocks of corresponding first-order gramians. CGA is ranked the third place, which considers controllabiltiy and observability in a single cross-gramian. SGA is in the last place, which includes controllability gramian only. Note that passivity is not preserved in either DGA or CGA. In the first case, the circuit matrix is symmetric but not positive semi-definite. In the second case, the projection is not a congruency transformation.

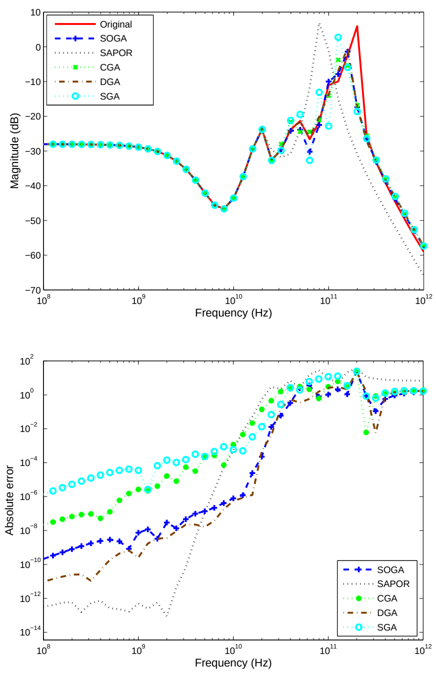

Figure 1: Comparison of different gramian approximation methods with SAPOR.

vspace-0.1in We then compare ETBR [4] with the similar Krylov subspace version approach IEKS [1]. The test circuit has 10000 node and 100 sources and the reduction order q is set to 6 for IEKS and the number of frequency samples used for ETBR is also set to 6. Fig. 2 shows the simulation results of ETBR and IEKS at the 200th node. The simulation errors compared with SPICE results are also shown in Fig. 2. We can see that ETBR is more accurate than IEKS over the entire simulation time.

6. CONCLUSION

In this paper, we review several recently proposed fast TBR methods based on gramian approximation techniques (SGA, DGA, SOGA, CGA, RGA) for model order reduction of RLCK circuits. We present the reduction methods based on both first-order formulation and second-order formulation. Experimental results show that those methods produce a better approximation over a wide frequency range than the corresponding Krylov subspace based methods in general.

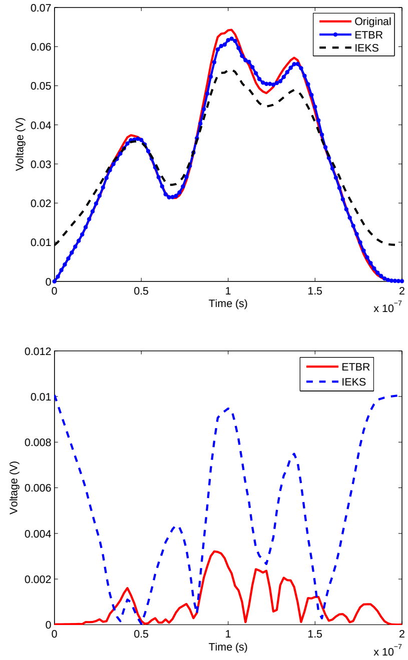

Figure 2: Accuracy comparison between ETBR and IEKS.

7. REFERENCES

[1] Y. Cao, Y. Lee, T. Chen, and C. C. Chen, "HiPRIME: hierarchical and passivity reserved interconnect macromodeling engine for RLKC power delivery," in *Proc. Design Automation Conf. (DAC)*, 2002, pp. 379–384.

[2] P. Feldmann and R. W. Freund, "Efficient linear circuit analysis by Pade approximation via the Lanczos process," *IEEE Trans. on Computer-Aided Design of Integrated Circuits and Systems*, vol. 14, no. 5, pp. 639–649, May 1995.

[3] R. W. Freund, "SPRIM: structure-preserving reduced-order interconnect macromodeling," in *Proc. Int. Conf. on Computer Aided Design (ICCAD)*, 2004, pp. 80–87.

[4] D. Li, S. X.-D. Tan, and B. McGaughy, "ETBR: Extended truncated balanced realization method for on-chip power grid network analysis," in *Proc. European Design and Test Conf. (DATE)*, 2008, pp. 432–437.

[5] B. Moore, "Principle component analysis in linear systems: Controllability, and observability, and model reduction," *IEEE Trans. Automat. Contr.*, vol. 26, no. 1, pp. 17–32, 1981.

[6] A. Odabasioglu, M. Celik, and L. Pileggi, "PRIMA: Passive reduced-order interconnect macromodeling algorithm," *IEEE Trans. on Computer-Aided Design of Integrated Circuits and Systems*, pp. 645–654, 1998.

[7] J. R. Phillips and L. M. Silveira, "Poor man's TBR: a simple model reduction scheme," *IEEE Trans. on Computer-Aided Design of Integrated Circuits and Systems*, vol. 24, no. 1, pp. 43– 55, 2005.

[8] B. N. Sheehan, "ENOR: model order reduction of RLC circuits using nodal equations for efficient factorization," in *Proc. Design Automation Conf. (DAC)*, 1999, pp. 17–21.

[9] Y. Su, J. Wang, X. Zeng, Z. Bai, C. Chiang, and D. Zhou, "SAPOR: second-order Arnoldi method for passive order reduction of RCS circuits," in *Proc. Int. Conf. on Computer Aided Design (ICCAD)*, 2004, pp. 74–79.

[10] S. X.-D. Tan and L. He, *Advanced Model Order Reduction Techniques in VLSI Design*. Cambridge University Press, 2007.

[11] B. Yan, S. X.-D. Tan, P. Liu, and B. McGaughy, "SBPOR: second-order balanced truncation for passive model order reduction of RLC circuits," in *Proc. Design Automation Conf. (DAC)*, 2007, pp. 158–161.

[12] B. Yan, S. X.-D. Tan, and B. McGaughy, "Second-order balanced truncation for passive model order reduction of RLCK circuits," *IEEE Trans. on Circuits and Systems*, 2008.

[13] H. Yu, Y. Shi, L. He, and D. Smart, "A fast block structure preserving model order reduction in inverse inductance circuits," in *Proc. Int. Conf. on Computer Aided Design (ICCAD)*, Nov. 2006, pp. 7–11.