Stochastic Extended Krylov Subspace Method for Variational Analysis of On-Chip Power Grid Networks [*]

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ABSTRACT

In this paper, we propose a novel stochastic method for analyzing the voltage drop variations of on-chip power grid networks with log-normal leakage current variations. The new method, called *StoEKS*, applies Hermite polynomial chaos (PC) to represent the random variables in both power grid networks and input leakage currents. But different from the existing Hermit PC based stochastic simulation method, extended Krylov subspace method (EKS) is employed to compute variational responses using the augmented matrices consisting of the coefficients of Hermite polynomials. Our contribution lies in the combination of the statistical spectrum method with the extended Krylov subspace method to fast solve the variational circuit equations for the first time. Experimental results show that the proposed method is about two-order magnitude faster than the existing Hermite PC based simulation method and more order of magnitudes faster than Monte Carlo methods with marginal errors. StoEKS also can analyze much larger circuits than the exiting Hermit PC based methods.

1. INTRODUCTION

Process-induced variability becomes the major design concern in the current 65 nm and coming 45nm VLSI technologies [18]. The process induced variations manifest themselves from wafer to wafer, die-to-die and device to device within a die [12, 11]. Some of the variations are systematic like those caused by lithograph process. Some are purely random like the doping density of impurities and edge roughness. As the technology moves to 65nm and comes near to 45nm, the variation will become more and more significant.

One important variation is the threshold voltage V_{th} variation. The reason is that subthreshold leakage current has a rapid increasing rate (about 5X-10X increase per technology generation [2]). Also subthreshold leakage current is highly sensitive to threshold voltage V_{th} variation, as the leakage current has exponential dependency on the threshold voltage V_{th}. If we model V_{th} as the random variable with Gaussian variation due to inter-die or intra-die process variations, the leakage currents will have a log-normal distribution as shown in [14]. As a result, leakage currents should be considered for variational analysis for important global interconnects like power grid networks.

A number of research work have been proposed recently to address the voltage drops variation issues in the on-chip power delivery networks under process variations. The voltage drop of power grid networks subject to the leakage current variations were first studied in [3, 4]. This method assumes that the log-normal distribution of the node voltage drop is caused by log-normal leakage current inputs and is based on a localized Monte Carlo (sampling) method to compute the variance of the node voltage drop. However, this localized sampling method is limited to the static DC solution of power grids modeled as resistor-only networks. Therefore, it can only compute the responses to the standby leakage currents. However, the dynamic leakage currents become more significant, especially when the sleep transistors are intensively used nowadays for reducing leakage powers. In [15, 13], impulse responses are used to compute the means and variances of node voltage responses due to general current variations. But this method needs to know the impulse response from all the current sources to all the nodes, which is expensive to compute for a large network.

Recently, a number of analysis approaches based on so-called statistical spectrum analysis method have been proposed for analyzing interconnect and power grid networks [16, 7, 9, 6]. This method is based on the orthogonal polynomial chaos expansion of random processes to represent and solve for the stochastic responses of linear systems. The orthogonal polynomial method, a.k.a statistical spectrum method, only needs to solve for some coefficients of the orthogonal polynomials by using normal transient simulation of the original circuits. Research work in [16] applied the statistical spectrum method to compute the variational delay of interconnects. In [7], statistical spectrum method has been applied to compute the voltage drop variations caused by Gaussian-only variations in the power grid wires and input currents (approximating them as Gaussian variations by using first-order Taylor expansion). Recent work [9] further considered the log-normal leakage variations applying the statistical spectrum method to solve for the variational voltage drops.

In this paper, we propose a new stochastic method for analyzing the voltage drop variations of on-chip power grid networks with log-normal leakage current variations. The new method, called *StoEKS*, still applies statistical spectrum method to solve for the variational responses. But different from the existing statistical spectrum-based simulation method, extended Krylov subspace method (EKS) is employed to compute variational responses using the augmented matrices consisting of the coefficients of Hermite polynomials. Our work is inspired by recent statistical spectrum-based model order reduction method [20]. We apply this work to the variational analysis of on-chip power grid networks considering the variational leakage currents with log-normal variation. We will show how the coefficients of Hermite PCs are computed for variational circuit matrices and the current moments used in EKS with log-normal distribution. Experimental results show that the proposed StoEKS is about two order magnitude faster than the existing Hermite PC based simulation method, having similar error compared with Monte Carlo method. StoEKS can analyze much larger circuits than the existing Hermite PC method in the same computation platform.

The rest of this paper is organized as the follows: Sec-

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tion 2 first presents the variational power grid models and problem we plan to solve. Section 3 reviews the orthogonal polynomial chaos based stochastic simulation method and improved extended Krylor Subspace method. Section 4 presents our new statistical power grid simulation method. Section 5 presents the experimental results and Section 6 concludes this paper.

2. PROBLEM FORMULATION

In this section, we first present the model of power grids we use in this paper. We then present the modeling issue of leakage current under intra-die variation. After this, we present the problem that we try to solve.

2.1 On-chip power grid network models

The power grid networks in this paper are modeled as RC networks with known time-variant current sources, which are obtained by gate-level logic simulations of the VLSI systems. For a power grid (versus the ground grid), some nodes have known voltages modeled as constant voltage sources. For C4 power grids, the known voltage nodes can be nodes inside the power grid. Given known deterministic vector of current sources, $u(t)$, the node voltages can be obtained by solving the following linear differential equation, which is formulated using modified nodal analysis (MNA) approach,

$$Gv(t) + C\frac{dv(t)}{dt} = Bu(t) \tag{1}$$

where G is the conductance matrix, C is the admittance matrix resulting from storage elements. $u(t)$ is a vector of time-varying node currents.

2.2 Variational power grid models

The G and C matrices and input currents $u(t)$ depend on the circuit parameters, such as metal wire width, length, metal thickness on power grids, and transistor parameters, such as channel length, width, gate oxide thickness, etc. Process-induced random variations can be classified into inter-die (die-to-die) variations and intra-die variations. In inter-die variations, all the parameters variations are correlated. The worst case corner can be easily found by setting the parameters to their range limits (mean plus 3σ). The difficulty lies in the intra-die variations, where the circuit parameters are not correlated or spatial correlated. Intra-die variations also consist of local and layout dependent deterministic components and random components. In this paper, we focus on the random variations, which typically are modeled as multivariate Gaussian process with any spatial correlation [1].

In this paper, we assume that we have a number of independent (uncorrelated) transformed ortho-normal Gaussian random variables $\xi_i(\theta), i = 1, ..., n$, which actually model the channel length and the device threshold voltage variations. The spatial correlation in the intra-die variation can be processed by using the principal component analysis method (or other methods like K-L transformation principal factor analysis etc.) to transform the correlated variables into un-correlated variables before spectral statistical analysis [7, 9]. Let Θ denote the process sampling space. Let $\theta \in \Theta$, $\xi_i : \theta \rightarrow R$ denote a normalized Gaussian variable and $\xi(\theta) = [\xi_{1d}(\theta), ..., \xi_{nd}(\theta)]$ is a vector of n Gaussian variable. After orthogonal transformation operation, we obtain independent random variable vectors $\xi = [\xi_1, ..., \xi_n]$. Notice that $n \leq r$ in general. Therefore, given the process variations, the MNA equation for (1) becomes

$$G(\xi)v(t) + C(\xi)\frac{dv(t)}{dt} = Bu(t, \xi) \tag{2}$$

Note that the input current vector, $u(t, \xi)$, follows the log-normal distribution and has both deterministic and random components. In this paper, we assume the dynamic currents (power) due to circuit switching are still modeled as deterministic currents as we only consider the leakage variations.

The problem we need to solve is to efficiently find the mean and variance of voltage $v(t)$ at any node at any time instance without using the time-consuming sampling-based method, such as Monte-Carlo.

3. REVIEW STATISTICAL SPECTRUM METHOD AND EXTENDED KRYLOV SUBSPACE METHOD

In the following, we briefly reviewed the spectral statistical based simulation with Hermite polynomial chaos used in [7, 9, 6].

3.1 Concept of Hermite polynomial chaos

Hermite PC utilizes a series of orthogonal polynomials (with respect to the Gaussian distribution) to facilitate stochastic analysis [19]. These polynomials are used as the orthogonal basis to decompose a random process in the similar way as sine and cosine functions are used to decompose a periodic signal in Fourier series expansion.

For a random variable $v(t, \xi)$ with limited variance, where $\xi = [\xi_1, \xi_2, ...\xi_n]$ is a vector of independent ortho-normal Gaussian random variables with zero mean. The random variable can be approximated by truncated Hermite PC expansion as follows [5]:

$$v(t, \xi) = \sum_{k=0}^{P} a_k H_k^n(\xi) \tag{3}$$

where n is the number of independent random variables, $H_k^n(\xi)$ is n-dimensional Hermite polynomials and a_k is the deterministic coefficient. The number of terms P is given by

$$P = \sum_{k=0}^{p} \frac{(n-1+k)!}{k!(n-1)!} \tag{4}$$

where p is the order of the Hermite PC. When one random variable is considered, the one-dimensional Hermite polynomials are expressed as follows:

$$H_0^1(\xi) = 1, H_1^1(\xi) = \xi, H_2^1(\xi) = \xi^2 - 1, H_3^1(\xi) = \xi^3 - 3\xi, ... \tag{5}$$

Hermite polynomials are orthogonal with respect to Gaussian weighted expectation (the superscript n is dropped for simple notation):

$$< H_i(\xi), H_j(\xi) >=< H_i^2(\xi) > \delta_{ij} \tag{6}$$

where δ_{ij} is the Kronecker delta and <*,*> denotes an inner product defined as follows:

$$< f(\xi), g(\xi) >= \frac{1}{\sqrt{(2\pi)^n}} \int f(\xi)g(\xi)e^{-\frac{1}{2}\xi^T\xi}d\xi \tag{7}$$

Like Fourier series, the coefficient a_k can be found by a projection operation onto the Hermite PC basis:

$$a_k(t) = \frac{< v(t, \xi), H_k(\xi) >}{< H_k^2(\xi) >}, \ \forall k \in \{0, ..., P\}. \tag{8}$$

3.2 Simulation approach based on Hermite PCs

Suppose that $v(t, \xi)$ is unknown random variable vector (with unknown distributions) like node voltages in (2), the coefficients can be computed by using Galerkin method, which states that the best approximation of $v(t,\xi)$ is obtained when the error $\Delta(t,\xi)$, which is defined as

$$\Delta(t,\xi) = G(\xi)v(t) + C(\xi)\frac{dv(t)}{dt} - Bu(t,\xi) \tag{9}$$

is orthogonal to the approximation. That is

$$< \Delta(t,\xi), H_k(\xi) >= 0,\ i = 0,1,...,P \tag{10}$$

In this way, we transform the stochastic analysis process to a deterministic process, where we only need to compute the coefficients of its Hermite PCs for responses we are interested in.

3.3 Extended Krylov subspace methods

In this subsection, we briefly review the extended Krylov subspace (EKS) method in [17] and [8] for fast computation of responses from linear dynamic systems.

The EKS method uses the Krylov-like reduction method to speedup the simulation process. Different from the Krylov-based model order reduction method, EKS performs the reduction considering both system matrices and input signals before the simulation. So it essentially is a simulation approach using Krylov subspace reduction method. It assumes input sources as piece-wise linear (PWL) sources.

Let $V = [\hat{v}_1, \hat{v}_2, ...\hat{v}_k]$ be an orthogonal basis for moment subspace $(m_0, m_1, ..., m_k)$ of the response $\mathbf{x}$. To get the basis V, we use following equations [17]

$$\hat{v}_i = \alpha_i(\prod_{j=0}^{i-1} \alpha_j m_i - \sum_{j=0}^{i-1} h_{i,j}\hat{v}_j) \tag{11}$$

where,

$$h_{i,k} = \hat{v}_k^T r_i, \alpha_i = \frac{1}{norm(\bar{v}_i)},\ \hat{\alpha}_i = \frac{\bar{v}_i}{norm(\bar{v}_i)} \tag{12}$$

$$\bar{v}_i = v_i - \sum_{j=0}^{i} h_{i,j}\hat{v}_i. \tag{13}$$

Then the original circuit described by (1) can be reduced to a smaller system

$$\hat{G}z + \hat{C}\frac{dz(t)}{dt} = \hat{B}u \tag{14}$$

where

$$\hat{G} = V^T G V, \hat{C} = V^T C V, \hat{B} = V^T B, x(t) = Vz(t)$$

After the reduced system in (14) has been solved for the given input $u(t)$, the solution $z(t)$ can then be mapped back into original space by $\tilde{x}(t) = Vz(t)$.

However, as the EKS models a PWL source as a sum of delayed ramps in Laplace domain, the terms contain $1/s$ and $1/s^2$ moments [17], while the tranditional Kyrlo space starts from $0th$ moment. Therefore, moment shifting must be made in EKS, which would cause complex computation and more errors. This problem is resloved in [8] in the IKES algorithm, which shows that the moments of $1/s$ and $1/s^2$ are zeros for PWL input sources.

As a result, with PWL sources $u(t)$ represented by a series of value-time pairs such as (a_1, τ_1), (a_2, τ_2),..., (a_{K+2}, τ_{K+2}), L moments needed to be calculated. As proposed in [8], the moments are calculated as

$$\begin{aligned} u_m \quad = \quad & (a_1 - \alpha_1\frac{\tau_1}{m+3})\beta_1^{(m+2)} - \sum_{i=1}^{k}(\alpha_i - \alpha_{i+1})\beta_{i+1}^{(m+3)} \\ & -(a_{K+2} - \alpha_{K+1}\frac{\tau_{k+2}}{m+3})\beta_{K+2}^{(m+2)},\ m = 1, ..., L.(15) \end{aligned}$$

Here,

$$u(s) = u_1 + u_2 s + u_3 s^2 + ... + u_L s^{L-1}$$

$$\beta_i^{(m)} = \frac{(-\tau_i)^m}{m!}, \alpha_i = \frac{a_{i+1} - a_i}{\tau_{i+1} - \tau_i}$$

4. NEW STOCHASTIC EXTENDED KRYLOV SUBSPACE METHOD – STOEKS

In this section, we present the new stochastic simulation algorithm, StoEKS, which is based on both the statistical spectrum method and the extended Krylov subspace method. The main idea is that we use the statistical spectrum method to convert the statistical simulation into a deterministic simulation problem. Then we apply EKS to solve the converted problem.

4.1 StoEKE algorithm flowchart

First, we present StoEKS algorithm flowchart, which is shown in Fig. 1. The algorithm starts with variational $G(\xi)$, $C(\xi)$ and variational input source $u(t,\xi)$. Then, it applies statistical spectrum method to convert the variational system (2) into a deterministic system, which consists of augmented matrices of $G(\xi)$ and $C(\xi)$ and position matrix B in (2) with corresponding to new unknowns. Then we use Hermite PC to represent the first L moments of current sources, U_L with log-normal distribution. Finally, we apply EKS/IEKS to solve the obtained deterministic system for response Z using the computed projection matrix V. After this we get back to the transient response of original augmented system by $x(t) = Vz(t)$. Finally we compute the mean and variance of any voltage nodes from $x(t)$.

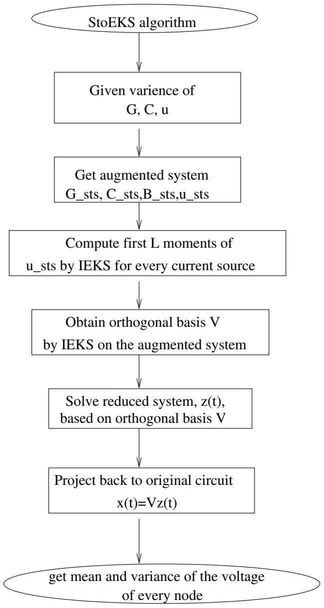

Figure 1: Flowchart of the StoEKS algorithm

In the following subsections, we present the detailed descriptions for some critical steps of the StoEKS algorithm.

4.2 Computation of the augmented circuit matrices

We first show how we convert the variational circuit equation into a deterministic one, which is suitable for EKS. Our work follows the recent proposed stochastic model order reduction (SMOR) method [20]. SMOR is based on Hermite Polynomial Chaos and the Krylov-based projection method.

Consider the variational power grid network system in (2) where $G(\xi)$ and $C(\xi)$ are represented in Hermite PC forms with a proper order:

$$\begin{aligned} G(\xi) &= G_0 + G_1\xi_1 + G_2\xi_2 + ... + G_N\xi_N \quad (16)\\ C(\xi) &= C_0 + C_1\xi_1 + C_2\xi_2 + ... + C_N\xi_N \\ u(t,\xi) &= u_0 + u_1\xi_1 + u_2\xi_2 + ... + u_N\xi_N \end{aligned}$$

where, η_i is the Hermite PC basis functions for $G(\xi)$, $C(\xi)$ and $u(t,\xi)$ and N is the number of these basis functions. Put (16) into (2), the system equation will be rewritten as [20]

$$\sum_{i=0}^{N}\sum_{j=0}^{N} G_i x_j \xi_i \xi_j + s\sum_{i=0}^{N}\sum_{j=0}^{N} C_i x_j \xi_i \xi_j = \sum_{i=0}^{N} u_i \xi_i \qquad (17)$$

After doing inner product of η_k on both sides of the equation (17), it would turn to

$$\begin{aligned} &\sum_{i=0}^{N}\sum_{j=0}^{N} G_i v_j E(\xi_i,\xi_j,\xi_k) + s\sum_{i=0}^{N}\sum_{j=0}^{N} C_i v_j E(\xi_i,\xi_j,\xi_k) \\ &= \sum_{i=0}^{N} u_i E(\xi_i,\xi_k), \ \ k = 0,1,...,N \qquad (18) \end{aligned}$$

where $E(\eta_i,\eta_j,\eta_k)$ is the inner product of the three Hermite PC basis functions η_i,η_j,η_k. The inner product is a constant and can be computed a priori and stored in a table for fast computation. Based on the N equations and the orthogonality of the Hermite polynomials, these equations can be written in matrix form as

$$(G_{sts} + sC_{sts})V = B_{sts}u_{sts} \qquad (19)$$

$$G_{sts} = \begin{bmatrix} G_{00} & \dots & G_{0N} \\ \vdots & \ddots & \vdots \\ G_{N0} & \dots & G_{NN} \end{bmatrix},$$

$$C_{sts} = \begin{bmatrix} C_{00} & \dots & C_{0N} \\ \vdots & \ddots & \vdots \\ C_{N0} & \dots & C_{NN} \end{bmatrix},$$

$$u_{sts} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_{N_u} \end{bmatrix}, V = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix}, B_{sts} = \begin{bmatrix} B_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & B_N \end{bmatrix} \quad (20)$$

$$B_i = B, G_{jk} = \sum_{i=0}^{N} G_i E(\eta_i,\eta_j,\eta_k), C_{jk} = \sum_{i=0}^{N} C_i E(\eta_i,\eta_j,\eta_k)$$

In [20], PRIMA-like reduction is performed on (19) to obtain the reduced variational system.

In the following, we consider a simple case where we only have three independent variables to illustrate the method. We assume that there are three indepedent variables ξ_g, ξ_c and ξ_I associated with matrices G and C and input sources respectively in the circuit. Then equation (2) becomes

$$G(\xi_g)v(t) + C(\xi_c)\frac{dv(t)}{dt} = Bu(t,\xi_I) \qquad (21)$$

The variation in width W and thickness T will cause variation in conductance matrix G and storage matrix C while variation in threshold voltage will cause variation in leakage currents $u(t)$. Thus, the resulting system can be written as [7]:

$$G(\xi_g) = G_0 + G_1\xi_g, C(\xi_c) = C_0 + C_1\xi_c \qquad (22)$$

G_0, C_0 represents the deterministic component of conductance and capacitance of the wires. G_1, C_1 represents sensitivity matrices of the conductance and capacitance . ξ_g, ξ_c are random variables with normalized Gaussian distribution, representing process variation in wires of conductance and capacitor, respectively. ξ_I is a normalized Gaussian distribution random variable representing variation in threshold voltage. $u(t,\xi_I)$ follows lognormal distribution as

$$u = e^{g(\xi_I)}, g(\xi_I) = \mu_I + \sigma_I\xi_I \qquad (23)$$

Using Gelerkin method as in [10] with second-order Hermite PCs, we end up solving the following equation

$$\widetilde{G}v(t) + \widetilde{C}\frac{dv(t)}{dt} = \widetilde{B}\widetilde{u}(t) \qquad (24)$$

where

$$\widetilde{G} = \begin{bmatrix} G_0 & G_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ G_1 & G_0 & 0 & 0 & 2G_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & G_0 & 0 & 0 & 0 & 0 & G_1 & 0 & 0 \\ 0 & 0 & 0 & G_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & G_1 & 0 & 0 & G_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & G_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & G_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & G_0 & 0 & 0 \\ 0 & 0 & 0 & G_1 & 0 & 0 & 0 & 0 & G_0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & G_0 \end{bmatrix}$$

$$\widetilde{C} = \begin{bmatrix} C_0 & 0 & C_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & C_0 & 0 & 0 & 0 & 0 & 0 & C_1 & 0 & 0 \\ C_1 & 0 & C_0 & 0 & 0 & 2C_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_1 & 0 & 0 & C_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & C_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & C_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & C_0 & 0 \\ 0 & 0 & 0 & C_1 & 0 & 0 & 0 & 0 & 0 & C_0 \end{bmatrix}$$

$$\widetilde{u}(t) = [u_0(t), u_1(t), u_2(t)]^T \qquad (25)$$

to get the mean and variance of node voltages in the circuit.

4.3 Computation of Hermite PCs of current moments with log-normal distribution

In this section, we show how to compute the variational current moments using Hermite PCs.

Assuming that the process variations in leakage current sources are $\vec{\xi} = [\xi_1, \xi_2, ..., \xi_n]$, following Gaussian distribution as

$$u(\vec{\xi}) \sim e^{\vec{\xi}} = e^{\sum_{j=0}^{n} g_j \xi_j} \qquad (26)$$

where $u(\vec{\xi})$ is a scalar value and represents one current source. The leakage current sources are therefore following lognormal distribution. We can then present $u(\vec{\xi})$ by using Hermite PC expansion form:

$$\begin{aligned} u(\vec{\xi}) &= \sum_{k=0}^{P} u_k H_k^n(\vec{\xi}) \\ &= u_0(1 + \sum_{i=1}^{n} \xi_i g_i + \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{(\xi_i\xi_j - \delta_{ij})}{< (\xi_i\xi_j - \delta_{ij})^2 >} g_i g_j \\ &\quad +...) \qquad (27) \end{aligned}$$

where

$$u_0 = e^{g_0 + \frac{1}{2}\sum_{i=1}^{n} g_i^2}, P = \sum_{k=0}^{p} \frac{(n-1+k)!}{k!(n-1)!} \qquad (28)$$

n is the number of random variables and k is the order of Hermite PC expansion.

As a result, the variational variable $u(t,\vec{\xi})$ leads to the input vector

$$u_{sts} = [u_1^T, u_2^T, ...u_P^T]^T \qquad (29)$$

in (19) which is obtained from equation (27).

However, in the EKS method, we need to compute the moments of input sources in frequency domain. Suppose

(a_{i1}, τ_{i1}), (a_{i2}, τ_{i2}),..., (a_{iK+2}, τ_{iK+2}) are PWL series of value-time pairs for u_i. Using equation (15), we can get the first L moments for each u_i, $i = 1, 2, ..., P$ in (29) respectively and we have

$$u_i(s) = m_{u_{i1}} + m_{u_{i2}}s+, ..., m_{u_{iL}}s^{L-1} \tag{30}$$

where $m_{u_{ik}}$ is the kth order moment vector of Hermite PCs coefficient for u_i. We compute the moments of Hermite PC coefficients for every current sources.

4.4 The StoEKS algorithm

Given the G_{sts}, C_{sts} and u_{sts} in moment forms, we can get the orthogonal V basis of these moments by equation (11). The reduced systems then can be obtained by these orthogonal basis V from equation(15). The reduced system will be

$$\hat{G}_{sts}z + \hat{C}_{sts}\frac{dz(t)}{dt} = \hat{B}_{sts}u_{sts} \tag{31}$$

Here,

$$\hat{G}_{sts} = V^T G_{sts} V, \hat{C}_{sts} = V^T C_{sts} V, \hat{B}_{sts} = V^T B_{sts} \tag{32}$$

The reduced system can be solved in the time domain by any standard integration algorithm. The solution of the reduced system, $z(t)$, can then be projected back to original space by $\tilde{X}(t) = Vz(t)$.

After getting $X(t)$ in the augmented matrix in(19), we can get mean and variance by

$$\begin{aligned} & E(x) \\ = \; & E(x_0 + \sum_{i=1}^{N_G} x_i\xi_i + \sum_{j=1}^{N_C} x_{j+N_G}\xi_j + \sum_{k=1}^{N_u} x_{j+N_G+N_C}\xi_k) \\ = \; & x_0 \qquad (33) \\ & var(x) \\ = \; & var(x_0 + \sum_{i=1}^{N_G} x_i\xi_i + \sum_{j=1}^{N_C} x_{j+N_G}\xi_j + \sum_{k=1}^{N_u} x_{j+N_G+N_C}\xi_k) \\ = \; & \sum_{i=1}^{N_G} x_i^2 var(\xi_i) + \sum_{j=1}^{N_C} x_{j+N_G}^2 var(\xi_j) \\ & + \sum_{k=1}^{N_u} x_{j+N_G+N_C}^2 var(\xi_k) \qquad (34) \end{aligned}$$

The mean and variance can be easily calculated by the characteristic of Hermite PC and the distrubition of ξ_1,ξ_2,...,ξ_N. Fig. 2 is the StoEKS algorithm for given G_{sts}, C_{sts}, B_{sts}, u_{sts}.

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Algorithms: StoEKS
Input: augmented system $G_{sts}$, $C_{sts}$, $B_{sts}$, $u_{sts}$
Output: the HPC coefficients of node voltage, X
1  Get the first $L$ moments of $u_{sts}$ for each current source
2  Compute the orthogonal basis of subspace from (19)
   V
3  Obtain the reduced system matrix from
   $\hat{G} = V^T G_{sts} V$, $\hat{C} = V^T C_{sts} V$, $\hat{B} = V^T B_{sts}$
4  Solve $\hat{G}z(t) + \hat{C}\frac{dz(t)}{dt} = \hat{B}u_{sts}(t)$
5  Project back to original space to get X(t) = Vz(t)
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Figure 2: StoEKS algorithm.

5. EXPERIMENTAL RESULTS

This section describes the simulation results of circuits with both capacitance, conductance variation and leakage current variation. The leakage current variation follows log-normal distribution. The capacitance and conductance variations follows Gaussian distribution. All the proposed methods have been implemented in Matlab 7.0. All the experimental results are carried out in a Linux system with dual Intel Xeon CPUs with 3.06Ghz and 1GB memory.

Fig.3 shows the node voltage distribution at one node of a ground network with 280 nodes, considering variation in conductance, capacitance and leakage current. The standard deviation of the log-normal current sources with one Gaussian variable is 0.1. The variance in conductance and capacitance are also 0.1. The mean and variance computed by the Hermite PC method, Hermite PC with EKS are also marked in the figure which fit very well with the MC results. In Fig. 3, the dotted lines are the mean and variance calculated by Monte Carlo. The solid lines are the mean and variance by the algorithm [9], which is named as *HPC*. The dashed line are the results from the new StoEKS. The Monte Carlo results are obtained by 3500 samples. Fig.4 shows the distribution at one node of a ground network with 2640 nodes. The parameter setup is the same as the parameters in the circuit with 280 nodes. The dotted lines represent the Monte Carlo results. And the dashed lines represent the results given by StoEKS. From these two figures, we can only see marginal difference between the three different methods.

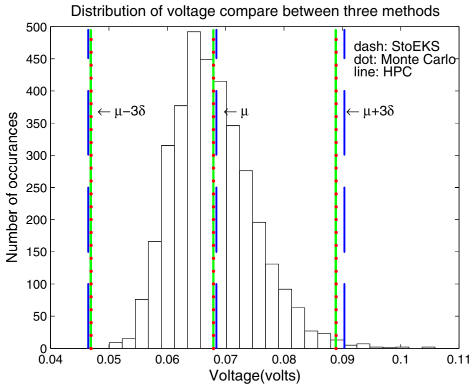

Figure 3: Distribution of the voltage in a given node by StoEKS, HPC and Monte Carlo

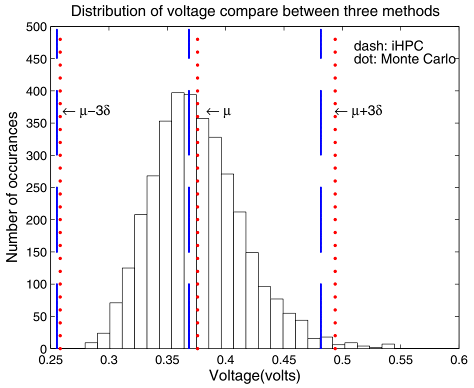

Figure 4: Distribution of the voltage in a given node by StoEKS and Monte Carlo

Table 1 shows the speedup of StoEKS and HPC method over Monte Carlo method. In the table, we observed that, it is unable to obtain the results from HPC when the circuit

Table 1: CPU time comparison of StoEKS and HPC with Monte Carlo methods

#node	MC	StoEKS	speedup	HPC [9]	speedup
280	1358	0.844	1609.39	73.4531	18.49
2640	18984	24.29	781.33	N/A	N/A
12300	1.37×10^5	486.92	282.96	N/A	N/A

Table 2: Accuracy Comparison of different methods, StoEKS, HPC and Monte Carlo

#node	Mean			Std Dev		
	MC	StoEKS	HPC	MC	StoEKS	HPC
280	0.067	0.068	0.068	0.007	0.0073	0.007
2640	0.376	0.368	N/A	0.039	0.037	N/A
12300	2.55	2.57	N/A	0.259	0.269	N/A

becomes large enough. Meanwhile, StoEKS can deliver all the results.

Table 2 and Table 3 show the mean and variance comparison of different methods with several circuits. Table 2 contains the values we obtain from different methods and Table 3 presents the error comparison based on *StoEKS* and *HPC* over Monte Carlo, respectively. We can see that our proposed method, StoEKS, only has marginal difference from Monte Carlo while it is able to perform simulation on much larger circuit than the existing *HPC* method on the same platform.

6. CONCLUSION

In this paper, we have proposed a fast stochastic method for analyzing the voltage drop variations of on-chip power grid networks. The new method, called *StoEKS*, applies Hermite polynomial chaos (HPC) to represent the random variables in both power grid networks and input leakage currents with log-normal distribution. We applied extended Krylov subspace method (EKS) to compute variational responses using the augmented matrices consisting of the coefficients of Hermite polynomials, which represents both variational parameters in circuit matrices and input sources. Experimental results have shown that the proposed method is about two order magnitude faster than the existing Hermite PC based simulation method and more order of magnitudes faster than Monte Carlo method with marginal errors. StoEKS also increases the analysis capacity of pervious statistical simulation methods based on the statistical spectrum method.

Table 3: Error comparison of StoEKS and HPC over Monte Carlo methods

#node	StoEKS % Error in μ	HPC % Error in μ	StoEKS % Error in σ	HPC % Error in σ
280	0.67	0.09	4.46	0.69
2640	2.12	N/A	4.15	N/A
12300	1.07	N/A	3.76	N/A

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