Reliability-Constrained Area Optimization of VLSI Power/Ground Networks Via Sequence of Linear Programmings

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Abstract

This paper presents a new method for determining the widths of the power and ground routes in integrated circuits so that the area required by the routes is minimized subject to the reliability constraints. The basic idea is to transform the resulting constrained nonlinear programming problem into a sequence of linear programs. Theoretically, we show that the sequence of linear programs always converges to the optimum solution of the relaxed convex problem. Experimental results demonstrate that the sequence-of-linear-programming method is orders of magnitude faster than the best-known method based on conjugate gradients, with constantly better optimization solutions.

1 Introduction

Power/Ground (p/g) networks connect the power/ground supplies within the circuit modules to the p/g pads on a chip. An important problem in p/g network design is to use the minimum amount of chip area for wiring power/ground networks while avoiding potential reliability failures due to electromigration and excessive IR drops. Of particular interest is the problem of p/g network optimization where the topologies of p/g networks are assumed to be fixed, and only the widths of wire segments are to be determined. Several methods have been developed to solve this problem [4, 5, 6, 7]. However, to the best of our knowledge, none of these methods have been incorporated into commercial CAD tools and used by industry.

One major obstacle is that these methods are based on constrained nonlinear programming, a process known to be computationally intensive (NP-hard [10]). These methods are applicable only to small size problems, while p/g networks in today's VLSI design may contain millions of wire segments (thus, variables). On the other hand, with the continuous shrinking of chip feature size, p/g network optimization is becoming increasingly important, since more and more portions of chip area are dedicated to power/ground routings, and the problem of IR drops and electro-migration deteriorates.

In this paper, we present a new method that can efficiently solve the power/ground optimization problem subject to reliability constraints. Our method is inspired by a key observation made by Chowdhury that if currents in wire segments are fixed, and voltages are used as variables, then the resulting optimization problem is convex [6]. However, instead of using the conjugate-gradient method as in [6], we show that the problem can be solved elegantly by a sequence of linear programs. We prove that there always exists a sequence of linear programs that converges to the optimal solution of the original convex optimization problem. Experimental results have demonstrated that usually a few linear programs are required to reach the optimal solution. With this, the complexity of the proposed method is proportional to the complexity of linear programming (which can be solved in polynomial time [10]). Therefore, our method is scalable, i.e., the CPU time increases approximately polynomially with the size of a network. In practice, we have observed that the new method is orders of magnitude faster than the conjugate gradient based method, with constantly better optimization results.

This paper is organized as follows. Section 2 presents some previous work. Section 3 describes the formulation of the p/g network optimization problem. The new method is presented in Section 4. Some practical considerations are described in Section 5. Experimental results from some large p/g networks are summarized in Section 6. Section 7 concludes the paper.

2 Previous Work

It is generally assumed that the average current drawn by the each module is known and is modeled as an independent current source (we do not consider the temporal correlations of current sources). The constraints from reliability and design rules include:

1. voltage IR drop constraints,
2. metal-migration constraints,
3. minimum width constraints.

The problem of determining the widths of wire segments of a P/G network to minimize the total p/g routing area subject to all these constraints is a constrained nonlinear optimization problem [4, 5].

In the method of Chowdhury and Breuer [4], resistance values and branch currents are selected as independent variables. Both the objective function and voltage IR drop constraints become nonlinear. The augmented Lagrangian method combined with steepest descent algorithm [3] is used to solve the resulting problem.

Dutta and Marek-Sadowska [7] used only resistance values as variables. All the constraints expressed in terms of nodal (terminal) voltages and branch currents, which have to be obtained by explicitly solving the electrical network, become nonlinear. The feasible direction method [2] is employed to solve the nonlinear problem. At each iteration step, extra effort is required to solve the electrical network for nodal voltages and branch currents, as well as their gradients by numerical differentiation.

Chowdhury [6] proposed a very interesting approach where both the nodal voltages and the branch currents are selected as variables. The optimization problem is solved iteratively in two stages. In the first stage, all the branch currents are fixed, and this leads to a convex programming problem solved by the conjugate gradient method [3]. In the second stage, all the nodal voltages are assumed fixed, and this gives rise to a linear programming problem. In comparison with other methods, this method is more general and more efficient. Unfortunately, the conjugate gradient method is not efficient enough to solve large size power/ground optimization problems arising in today's VLSI design.

A method proposed by Mitsuhashi and Kuh [9] further extends power/ground network optimization to include topology selection. In this paper, we assume that the topology is fixed.

3 Problem Formulation and General Optimization Procedure

Our work follows the formulation and the general optimization procedure of Chowdhury [6]. His results are reviewed briefly in this section.

3.1 Problem Formulation

Let $G = \{N, B\}$ be a p/g network with n nodes $N = \{1, ..., n\}$ and b branches $B = \{1, ..., b\}$. Each branch i in B connects two nodes: $i1$ and $i2$ such that the positive current flows from $i1$ to $i2$. Let l_i and w_i be the length and width of branch i. Let ρ be the sheet resistivity. Then the resistance r_i of branch i can be expressed as:

$$r_i = \frac{V_{i1} - V_{i2}}{I_i} = \rho \frac{l_i}{w_i}. \quad (1)$$

The total p/g routing area, which is the objective function to be minimized, can be expressed as

$$f(\mathbf{V}, \mathbf{I}) = \sum_{i \in B} l_i w_i = \sum_{i \in B} \frac{\rho I_i l_i^2}{V_{i1} - V_{i2}}. \quad (2)$$

Instead of using widths w_i, $i \in B$, as variables, we choose to solve branch current I_i and nodal voltage V_{i1} and V_{i2}. The constraints needed to satisfy are as follows.

1. **The voltage IR drop constraints.** To ensure the correct and reliable logic operation, the nodes connected to a p/g pad should have the voltage values close to the value of the pad:

$$\begin{aligned} V_j &\geq V_{j,min} \text{ if } j \text{ is connected to a power pad.} \\ V_j &\leq V_{j,max} \text{ if } j \text{ is connected to a ground pad.} \end{aligned} \quad (3)$$

   where $V_{j,min}$ and $V_{j,max}$ are given constants.

2. **The minimum width constraints.** The widths of the p/g segments are technologically limited to the minimum width allowed in the layer where the segment lies. Thus, we have

$$w_i = \rho \frac{l_i I_i}{V_{i1} - V_{i2}} \geq w_{i,min}, \quad (4)$$

   where $w_{i,min}$ are given constants.

3. **The current density constraints (electro-migration).** Electro-migration on a p/g wire sets an upper bound on the current density of the p/g segment [1]. For a fixed thickness σ of a layer, this constraint for branch i can be expressed as:

$$|I_i| \leq w_i \sigma. \quad (5)$$

   It can be re-written as the following nodal voltage constraint:

$$|V_{i1} - V_{i2}| \leq \rho l_i \sigma. \quad (6)$$

4. **Kirchoff's current law (KCL).**

$$\sum_{i \in B(j)} s_i I_i = 0, \quad (7)$$

   for each node $j = \{1, ..., n\}$ and $B(j)$ is the set of indices of branches connected to node j and s_i is 1 if current direction for branch i is toward node j and -1 otherwise.

P/g network optimization is to minimize (2) subject to constraints (3), (4), (6) and (7). It will be referred to as Problem **P**. Problem **P** is a constrained nonlinear optimization problem.

3.2 General Optimization Procedure

To reduce the complexity of solving Problem **P**, Chowdhury proposed a relaxed optimization procedure:

- Start with an initial feasible solution of **P**.
- Assume that all branch currents are fixed. Then the objective function can be rewritten as

$$f(\mathbf{V}) = \sum_{i \in B} \frac{\alpha_i}{V_{i1} - V_{i2}}, \tag{8}$$

where $\alpha_i = \rho I_i l_i^2$, We further restrict the changes of nodal voltages such that their current directions do not change during the optimization process. i.e.,

$$\frac{V_{i1} - V_{i2}}{I_i} \geq 0. \tag{9}$$

With this, the minimum width constraints (4) can be re-written as

$$\frac{V_{i1} - V_{i2}}{I_i} \leq \frac{\rho l_i}{w_{i,min}}. \tag{10}$$

This leads to a simplified optimization problem: find all nodal voltages to minimize (8) subject to constraints (3), (6), (9), and (10). Let us denote it as problem **P1**.

- Assume that all nodal voltages are fixed. Then the objective function can be re-written as:

$$f(\mathbf{I}) = \sum_{i \in B} \beta_i I_i, \tag{11}$$

where $\beta_i = \frac{\rho l_i^2}{V_{i1} - V_{i2}}$.
Similarly, we restrict the changes of current directions during the optimization process. i.e.,

$$\frac{I_i}{V_{i1} - V_{i2}} \geq 0. \tag{12}$$

Then problem **P** reduces to the following problem: find all the branch currents to minimize (11) subject to (4), (7) and (12). Let us denote this problem as **P2**.

Chowdhury showed that problem **P1** can be converted to an un-constrained convex programming problem and solved by the conjugate gradient method. **P2** is a linear programming problem. Therefore, solving **P** is to start with an initial feasible solution, then iteratively solve **P1**, then **P2**.

4 New Linear-Programming Based Algorithm

The new method uses a sequence of linear programmings to solve the nonlinear programming problem **P1**. In this section, we present the new method and prove that it always converge to the optimum solution of the relaxed problem **P1**.

The basic idea is to linearize the nonlinear objective function (8). To see this, we further define branch voltage drop variable as $v_i = sign(I_i)(V_{i1} - V_{i2})$ for each branch i, where $sign(x) = 1$ if $x > 0$ and $sign(x) = -1$ if $x < 0$. Note that $v_i \geq 0$. Then in terms of $v_i, i = 1, ..., b$, the objective function (8) can be expressed as

$$f(\mathbf{v}) = \sum_{i \in B} \frac{|\alpha_i|}{v_i}, \tag{13}$$

where $\mathbf{v} = \{v_1, v_2, ..., v_b\}^T$. Suppose that we have an initial feasible solution $\mathbf{V}^0$ and corresponding $\mathbf{v}^0$ satisfying all the constraints. We then take the Taylor's expansion of $f(\mathbf{v})$ around $\mathbf{v}^0$ and keep only the constant and linear terms. The resulted objective function is called $g(\mathbf{v})$,

$$g(\mathbf{v}) = f(\mathbf{v}^0) + \frac{\partial f(\mathbf{v}^0)}{\partial \mathbf{v}}(\mathbf{v} - \mathbf{v}^0) = \sum_{i \in B} \frac{2|\alpha_i|}{v_i^0} - \sum_{i \in B} \frac{|\alpha_i|}{v_i^{0^2}} v_i. \tag{14}$$

Instead of minimizing $f(\mathbf{v})$, we minimize the $g(\mathbf{v})$ as long as these two functions satisfy the following property:

$$g(X) > g(Y) \Longrightarrow f(X) > f(Y), \tag{15}$$

where $\Longrightarrow$ means *imply*. This requirement essentially says that as long as we reduce $g(X)$ we always can reduce $f(X)$.

To ease our analysis, we first take a look at each individual product term in the objective function (13), which has the following form $h(x) = c/x, x > 0$, where c is a constant and also $c > 0$. Fig. 1 draws the function $h(x) = c/x$ with $c = 1$ and its linearized first-order Taylor's expansion function $H(x)$ at expansion point $x_0 = 0.04$. We note that both

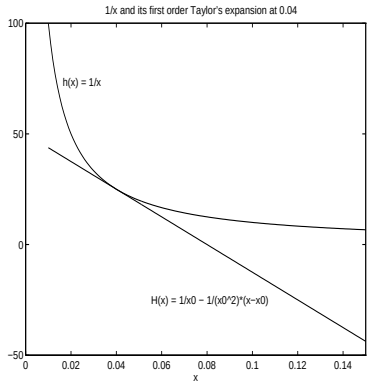

Figure 1: h(x) = 1/x and its first-order expansion at 0.04.

$h(x)$ and $H(x)$ are monotonical decreasing functions in x in the range $(0, \infty)$ with the property that $h(x) > H(x)$.

Back to $f(\mathbf{v})$ and $g(\mathbf{v})$, we have the following two optimization scenarios:

1. If all the branch voltage drops v_i increase after the optimization, we have $f(\mathbf{v}^0) > f(\mathbf{v})$ and $g(\mathbf{v}^0) > g(\mathbf{v})$. Because all the product terms in both $f(\mathbf{v}^0)$ and $g(\mathbf{v}^0)$ monotonically decrease as each v_i increases, the property (15) is always satisfied.

2. If only some branch voltage drops increase and others decrease or stay unchanged, then the property (15) may not be satisfied due to the fact that for $x < x_0$, $h(x)$ will increase very quickly, while $H(x)$ only increase linearly. As a result, we may end up with $f(\mathbf{v}) > f(\mathbf{v}^0)$ while $g(\mathbf{v}) < g(\mathbf{v}^0)$.
   In this case, we can limit the solution space to the neighborhood of v^0 such that the relation property (15) holds by imposing the following constraint for each branch i:
$$\xi v_i^0 \leq v_i \leq (2-\xi)v_i^0, \tag{16}$$
   where ξ is called the *restriction factor*, $0 < \xi < 1$. Note that we can always achieve the relation (15) by choosing $\mathbf{v}$ to be close enough to the $\mathbf{v}^0$ (ξ is close enough to 1) as $g(\mathbf{v})$ is essentially the first-order approximation of $f(\mathbf{v})$.

On the other hand, any increase in any branch voltage drop $v_i, i \in \{1,...,b\}$ always decreases both $f(\mathbf{v})$ and $g(\mathbf{v})$. This implies that the upper bound in (16) is not necessary and we can combine the solution space, where relation (15) holds in both two scenarios, into a single space:

$$\xi v_i^0 \leq v_i. \tag{17}$$

Back to the nodal voltages, the linearized objective function and the restriction constrain can be rewritten as:

$$g(\mathbf{V}) = \sum_{i\in B} \frac{2\alpha_i}{(V_{i1}^0 - V_{i2}^0)} - \sum_{i\in B} \frac{\alpha_i}{(V_{i1}^0 - V_{i2}^0)^2}(V_{i1} - V_{i2}), \tag{18}$$

$$\xi sign(I_i)(V_{i1}^0 - V_{i2}^0) \leq sign(I_i)(V_{i1} - V_{i2}). \tag{19}$$

Note that the constraint (19) does not require the nodal voltage $V_{ix}, i \in \{1,...,n\}$, to be close to their initial values since only the relative differences among them matter. Because the constraint (19) already implies the constraint (9), only (19) remains in the optimization of solving for nodal voltages. Therefore, we have the following optimization problem, denoted as **P3**: minimize (18) subject to (3), (6), (10), and (19). Problem **P3** is a linear programming problem. For convenience, we use Γ to denote the feasible solution space of problem **P3** defined by (3), (6), (10), and (19). We use Ω to denote the feasible solution space of problem **P1** as defined by constraints (3), (6), (10), and (9). Clearly $\Gamma \subseteq \Omega$.

The procedure for solving problem **P2**, i.e., solving for nodal voltages, can be transformed to the problem of choosing ξ and solving **P3**, and then repeating this process until the optimum solution is found. This process is illustrated in Fig. 2. Now we are ready to summarize the entire optimization procedure as follows:

New Power/Ground Optimization Algorithm

1. Analyze the network G to obtain the initial $\mathbf{V}^k, \mathbf{I}^k$ for $k = 0$.

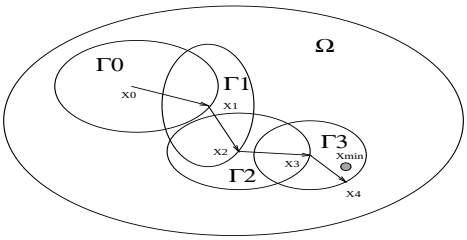

Figure 2: An illustration of sequence of linear programmings.

2. Construct the minimum width constraints (10), current density constraints (6) and additional constraints (19) using $\mathbf{I}^k$.

3. Minimize $g(\mathbf{V}^k)$ subject to constraints (3), (6), (10) and (19) by linear programming, record the result as $\mathbf{V}_l^k$, where l begins from 1. If $f(V_l^k) \geq f(V_{l-1}^k)$, increase ξ and repeat substep l. Construct the constraint (19) with the result $\mathbf{V}_l^k$ and repeat step 3 until no improvement can be found and record the result from the last iteration l in step 3 as $\mathbf{V}^{k+1}$.

4. Construct the minimum width and its companion constraint (4) and (12) using $\mathbf{V}^{k+1}$ for each branch.

5. Minimize the objective function (11) subject to the constraints (4), (12) and (7) by linear programming and record the result as $\mathbf{I}^{k+1}$.

6. If $|f(V^{k+1}, I^{k+1}) - f(V^k, I^k)| < \varepsilon$, ε is the termination criterion, then stop, otherwise k = k + 1 and goto step 2.

Since $f(\mathbf{V})$ is a convex function [6] under the fixed branch currents, any local minimum of $f(\mathbf{V})$ is also the global minimum. We have the following proposition:

Proposition 1 *There exists a ξ so that step 3 always converges to the global minimum in Ω.*

Proof Sketch: For function $f(X) = \sum_{i=1}^N \frac{1}{x_i}$, the truncated linear Taylor expansion around X_0 is $g(X) = \sum_{i=1}^N (\frac{1}{x_{0i}} - \frac{x_i - x_{0i}}{x_{0i}^2})$. We have a mathematical proof that for each X_0, there is a non-empty vicinity Γ of X_0 (by setting ξ close to 1) such that for each $X \in \Gamma$ if $g(X) < g(X_0)$ then $f(X) < f(X_0)$.

In other words, given an initial point $X_0 = V_l^k$, one can always find a non-empty vicinity of X_0, by increasing ξ close enough to 1, such that minimizing the linear expansion g at V_{l+1}^k, also decreases the original function f.

So a decreasing sequence $\{V_1^k, V_2^k, .., V_l^k\}$ is generated, which guarantees to converge to a local minimum. Since $f(X)$ is a convex function, the local minimum is also the global minimum, so the new algorithm will converge to the global minimum in Ω. □

We note that a similar technique, called *successive linear programming (SLP)* [3], was first proposed by Griffith and Stewart to solve problems in oil and chemical industries [8].

5 Practical Considerations and Algorithmic Remarks

In this section, we describe some practical considerations on how to apply the proposed method to optimize power/ground networks in practice.

Selection of restriction factor ξ. An important issue is how to select the restriction factor ξ. In general, as long as property (15) holds, Γ should be as large as possible in order to reduce the number of required iterations. In our implementation, we always begin with small ξ to maximize Γ. If property (15) is not satisfied, then we increase ξ such that property (15) can be satisfied again.

Algorithm Scalability. In practice, the number of linear programmings needed to reach the optimum solution is only a few. Thus the time complexity of our method is proportional to the complexity of linear programming. It is known that linear programs can be solved in polynomial time using the interior point method [3]. This makes our method very promising for optimizing very large VLSI p/g networks.

Input Data Scaling. For practical P/G networks, the module currents are usually in the range of $1 \times 10^{-9} A$. Branch currents and branch voltages could become very small if no scaling is used. This would give rise to some numerical problems for the linear programming solver. In our implementation, scaling is used.

Converting power networks to ground networks. Power networks should be transferred into ground networks to further improve the numerical stability. This is due to the fact that the voltage drop close to zero can be represented more precisely than the voltage drop close to other constant value. For example, the voltage drop 2.5×10^{-5} has to be represented by 4.999975 if the source voltage is 5 volt. It can be proven that a power network can be transferred into a ground network by using the following rules:

1. short-circuit all the VDD pads to the ground.
2. inverse the directions of all the independent current sources.

6 Experimental Results

A CAD tool for p/g network optimization has been developed based on the proposed sequence-of-linear-programming method. For comparison, Chowdhury's conjugate gradient method [6] has been carefully implemented[1]. A set of p/g networks with ten to more than ten thousand segments has been tested. All experiments are performed on a SUN Ultra I workstation with 169MHz clock rate. The results are summarized in Table 1.

[1]In fact, our research was motivated by initial attempt to implement the conjugate gradient method for power/ground network optimization in an industry setting.

In Table 1, columns 1 to 3 list, respectively, the circuit name (*ckt*), the number of nodes in the circuit (*# node*) and the number of branches (*# bch*). The number of iterations (# iter) (solving **P1-P2**) , CPU time in seconds (*time*), and the reduced chip area of the original area in percentage (*area reduced (%)*) are reported in columns 4, 5 and 6 for the new algorithm and columns 7, 8 and 9 for the conjugate gradient method. For example, for p/g network *p4x4*, the new method reduces the chip area used by 95.1%, while the conjugate gradient method reduces the chip area used by 94.2%.

We have the following observations:

- For large p/g network optimization *p3x500, g300x10, p100x100*, the conjugate gradient method finds solutions that use much more chip areas than that of the new method. This is due to the numerical problem inherent in the conjugate gradient method. During the course of optimization, the conjugate directions may deteriorate such that the algorithm gets stuck at a place far away from the optimum solution. In our implementation, we reinitialize the direction vector after *n* steps, where *n* is the number of nodes in the p/g network. Due to the large value of *n* for large p/g networks, the conjugate directions may deteriorate more, and thus prevents the algorithm from reaching the optimum. However, if we reinitialize the directions more frequently, the quality of the solution can be improved at the expense of more CPU time.
- The new algorithm is orders of magnitude faster than the conjugate gradient method.
- We have observed that for all the p/g networks tested, with one iteration, the new algorithm is able to reduce the most of the silicon cost that can be reduced, i.e., finds a solution that is very close to the optimum. Fig. 3 shows how the objective function (overall silicon cost) reduces with the number of iterations for an example network.

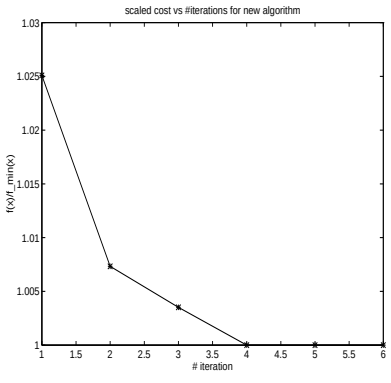

Figure 3: The cost reduction with the number of iterations.

Table 1: Comparison of the new algorithm against the conjugate gradient method.

ckt	# node	# bch	new algorithm			conjugate method			Speedup
			# iter	time	area reduced(%)	# iter	time	area reduced(%)	
p4x4	17	23	4	0.43	95.1	21	78.7	94.2	183.0
g20x20	402	439	3	12.6	91.8	255	36147.1	90.8	2868.8
p3x500	1502	1505	2	37.6	47.8	67	2135.4	26.8	>56.8
g300x10	3002	3599	2	609.9	93.7	137	15192.1	78.4	>25.0
p100x100	10002	10199	4	1325.6	80.7	117	41716.8	48.9	>31.5

In the experiments above, restriction factor ξ is set to 0.85. Recall that the feasible region Γ for linear program **P3** is controlled by ξ. The closer ξ to 1, the smaller the region Γ, and the better $g(\mathbf{V})$ approximates $f(\mathbf{V})$. That implies, the total number of linear programmings (solving **P3**) required for solving **P1** will increase, but the chance to find the minimum solution of the original problem increases. On the other hand, if we reduce ξ more close to 0, the feasible solution region Γ of the linear problem **P3** enlarges. Then the linearized function $g(\mathbf{V})$ may not be able to approximate well the original function $f(\mathbf{V})$. As a consequence, the sequence of linear programs may converge to a solution with the cost higher than the optimum one. This observation has been confirmed by the experiment on the example circuit *p4x4* as shown in Fig. 4 and 5.

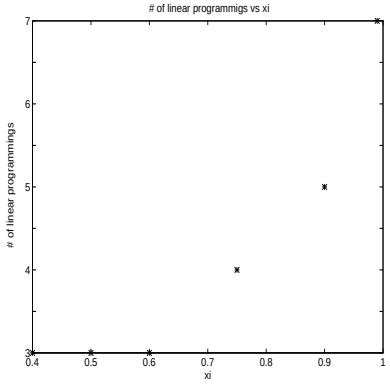

Figure 4: The number of linear programmings versus ξ (*xi*).

7 Conclusions

A new method is proposed for determining the widths of the wire segments in a power/ground network so that the chip area required by the power/ground network is minimized while ensuring voltage IR drops and electromigration constraints. We have shown, theoretically, that the new method is capable of finding a solution as good as that by the best-known method. Experimental results have demonstrated that the proposed method is orders of magnitude faster than the best-known method with constantly *better* solution quality.

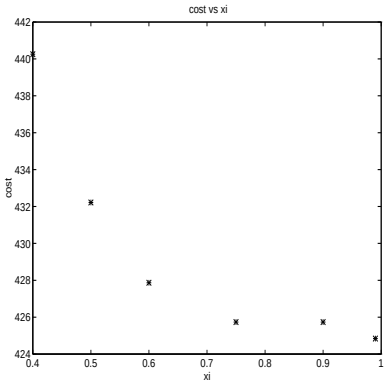

Figure 5: The final cost versus ξ (*xi*).

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