

The fast Convergence Analysis of Finite-time Consensus for Multi-agent with Switching Topology

LI Rui^{1,3}, SHI Yingjing², Sheldon X.-D. Tan³

1. School of Automation, University of Electronic Science and Technology of China, Chengdu 611731, P. R. China
E-mail: hitlirui@gmail.com

2. Institute of Astronautics and Aeronautics, University of Electronic Science and Technology of China, Chengdu 611731, P. R. China
E-mail: shiyingjing@gmail.com

3. Department of Electrical Engineering, University of California at Riverside, Riverside, CA 92521, U.S.
E-mail: stan@ee.ucr.edu

Abstract: In this paper, a new consensus problem of multi-agent systems with switching topology in finite-time domain is investigated. The problem can be formulated as a time-optimal control problem of a switched system with unknown weight parameters and switching times. By applying a time transform method and an inequality transform technique, we transform the fast convergence consensus into a canonical optimal parameter selection problem, which can be solved by a nonlinear optimization method. One illustrative example is given to show the effectiveness of the proposed method.

Key Words: Finite-time consensus, Multi-agent, Time scaling transform, Optimization method

1 Introduction

Recently, distributed cooperative control for multi-agent systems has received significant attention due to its potential applications in various areas such as formation control [1, 2], flocking problem [3, 4], proportional task allocation [5] etc. As a typical collective behavior, the consensus of multi-agent has become an active research topic. For example, the consensus problems of multi-agent systems with switching topology and directional communications were studied in [6–9]; The optimality properties of consensus algorithms were investigated in [10–13]. A comprehensive survey can be found in [14] and [15].

As practical situations create demands for faster convergence and better disturbance rejection, it is often required that consensus can occur in finite time. Therefore, finite-time consensus is more appealing and has attracted the attention of researchers. In [16], Wang and Xiao presented one framework for constructing finite-time consensus protocols, which were given as the continuous state feedback form for both the bidirectional interaction case and the unidirectional interaction case. Further, Xiao and Wang et al. [17] gave some finite-time consensus results for the case of unidirectional and intermittent links. In [18], Jiang et al. analytically established the explicit expression of the consensus state for the entire group and proved that the agents of the group under a particular type of nonlinear interaction can reach the consensus state in finite time with fixed and switching undirected topologies. In [19], Sayyaadi et al. also presented a simple distributed continuous-time protocol to guarantee finite-time consensus in weighted directed/undirected and fixed/switching networks. The stability of the system and the solvability of the consensus algorithm were proved for network topologies containing a spanning tree. In [20], Khoo et al. investigated leader-follower finite-time consensus control of multi-agent networks with input disturbances by using terminal sliding mode control scheme. A new terminal sliding

mode surface was proposed to guarantee finite-time consensus under fixed topology. In [21], Wang et al. presented a finite-time control law for continuous multi-agent system moving with obstacle avoidance, which ensures that all the agents can pass the obstacles on their way and archived consensus of relative position in finite time.

Although much work has been done for finite-time consensus problem and optimality issue in consensus algorithms has also been studied in literature, the existing methods seldom consider the optimal consensus problem for multi-agent systems in finite time domain. In this paper, we investigate the fast convergence consensus (FCC) problem for finite-time multi-agent systems. Usually, with moving of the network, the topology structure of the multi-agent system is dynamic. Therefore, we focus on the multi-agent system with the switching topology in this paper. We first model the FCC problem as an optimal control problem with the continuous state inequality constraints, where the weight parameters and the switching points of the topology structure need to be determined to achieve the minimum convergence time by using the given consensus protocol. Due to the variable switching points and the continue state inequation constraints, the classical optimization method is difficult to solve. As a result, a time transform method called time scaling transform (TST)^[22], is then used to map the variable switching times into fixed points and a constraint transcription method^[23] is applied to approximate the continue state inequality constraints by a sequence of standard inequality constraints. Thus we end up with a series of canonical optimal parameter selection problems, which can be solved by nonlinear optimization techniques.

The remainder of this paper is organized as follows. In section 2, some concepts in graph theory and finite-time consensus problems are introduced. Then the fast convergence consensus problem for finite-time multi-agent system is formulated in Section 3. In section 4, we develop the parameterization optimization method to solve the FCC problem. In section 5, a numerical example is given to illustrate our results. Finally, we draw conclusions in Section 6.

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2 Background and Definition

To solve the coordination problems and to model the information exchange between agents, graph theory is introduced first. Let $G = \{V, \varepsilon, A\}$ be a weighted digraph, where $V = \{v_1, v_2, \dots, v_n\}$ is the vertex set, and vertex v_i corresponds to agent i . $\varepsilon(G) \subset \{(v_i, v_j) : i, j \in I_n\}$ is the set of edges, where $I_n = \{1, 2, \dots, n\}$. The set of neighbors of vertex v_i is denoted by $N_i = \{v_j : (v_j, v_i) \in \varepsilon\}$. In an undirect graph, $(i, j) \in \varepsilon$ implies $(j, i) \in \varepsilon$. The weighted adjacency matrix A is defined as $a_{ij} > 0$ if $(j, i) \in \varepsilon$, and $a_{ij} = 0$ otherwise. We assume $a_{ii} = 0$ for all $i \in I_n$.

Let x_i denote the state of agent i , $i \in I_n$, and suppose that agent i possesses the following dynamics

$$\dot{x}_i(t) = u_i(t) \quad (1)$$

with the initial condition:

$$x_i(t_0) = x_i^0, \quad (2)$$

where u_i is the state of the feedback, called protocol, to be determined by the state information received by agent i from its neighbors. We say that nodes of a network have reached consensus if and only if $x_i = x_j$ for all $i, j \in I, i \neq j$. It is said that a protocol u_i solve the finite-time consensus problem if for any initial states, there exist a time t^* and a real number κ such that $x_j(t) = \kappa$ for $t \geq t^*$ and for all $j \in I_n$ ^[16]. A common finite-time consensus algorithm studied in [16] is given as follows:

$$u_i = \sum_{j \in N_i} a_{ij} \text{sign}(x_j - x_i) |x_j - x_i|^{\alpha_{ij}}, \quad (3)$$

where $0 < \alpha_{ij} < 1$, $i \neq j$, $|\cdot|$ represents the absolute value, and $\text{sign}(\cdot)$ is the sign function defined by

$$\text{sign}(r) = \begin{cases} 1, & r > 0 \\ 0, & r = 0 \\ -1, & r < 0. \end{cases}$$

In [16], the following convergence theorem is discussed.

Theorem 1 *Suppose that $G(A(t))$ is undirected and the sum of time-intervals, in which $G(A(t))$ is connected, is sufficient large. If $\alpha_{ij}(t) = \alpha_{ji}(t)$ and $0 < \alpha_{ij}(t) < 1$ for all i, j, t , then protocol (3) solves the finite-time average-consensus problem.*

3 Problem Formulation

Since the convergence speed is an important issue for finite-time consensus, in the following we will discuss the fast convergence consensus problem, i.e., find the control protocol, such that the system state arrives consensus within the minimum time.

If the topology structure is fixed, the parameters designed in control protocol (3) include $\{a_{ij}\}$ and $\{\alpha_{ij}\}$, $j \in N_i$, and $i \in I_n$. The FCC problem with fixed topology can be posed as the following time optimal control problem:

Problem 1 *Given the multi-agent system (1) and the initial condition (2), find parameter $\{a_{ij}\}$ and $\{\alpha_{ij}\}$, $j \in N_i$ in consensus protocol u_i , $i = 1, 2, \dots, n$, such that the cost*

function

$$J = t_f + \int_{t_0}^{t_f} \sum_{i=1}^n u_i^2(t) dt \quad (4)$$

is minimized subject to the following constraints

$$a_{ij} > 0 \quad (5a)$$

$$0 < \alpha_{ij} < 1 \quad (5b)$$

$$a_{ij} = a_{ji} \quad (5c)$$

$$\alpha_{ij} = \alpha_{ji}, \quad (5d)$$

where u_i is given in equation (3).

Problem 1 is an optimal parameter selection problem, which can be solved by some modified optimization method. In this paper, we are interested to investigate the optimal condition in case of a network with switching topology. In fact, Problem 1 can be viewed as a special case of FCC problem with the switching topology.

Let N be a given integer. Let the time period $[t_0, t_f]$ is partitioned into N subintervals with the $N + 1$ partition points which are denoted by t_0, t_1, \dots, t_N with $t_{k-1} < t_k$ and $t_N = t_f$. Define

$$\Upsilon = \tau = [t_1, t_2, \dots, t_N]^T \in R^N. \quad (6)$$

We now approximate the control protocol (3) as follows:

$$u_i = \sum_{j \in N_i} \sum_{k=1}^N a_{ij}^k \text{sign}(x_j - x_i) |x_j - x_i|^{\alpha_{ij}^k} \chi_{[t_{k-1}, t_k)}(t), \quad (7)$$

where $\chi_{[t_{k-1}, t_k)}(t)$ denotes the indicator function of the interval $[t_{k-1}, t_k)$ described by

$$\chi_I(t) = \begin{cases} 1 & t \in I \\ 0 & \text{elsewhere.} \end{cases}$$

Let

$$\alpha_i^l = [\alpha_{i1}^l, \dots, \alpha_{i(i-1)}^l, \alpha_{i(i+1)}^l, \dots, \alpha_{in}^l]^T \quad (8)$$

$$\alpha_i = [(\alpha_i^1)^T, (\alpha_i^2)^T, \dots, (\alpha_i^N)^T]^T \quad (9)$$

and

$$\Lambda = \{\alpha | \alpha = [\alpha_1, \alpha_2, \dots, \alpha_n]^T \in R^M\}, \quad (10)$$

where

$$M = N \cdot n(n-1). \quad (11)$$

Similarly, let

$$\gamma_i^l = [a_{i1}^l, \dots, a_{i(i-1)}^l, a_{i(i+1)}^l, \dots, a_{in}^l]^T \quad (12)$$

$$\gamma_i = [(\gamma_i^1)^T, (\gamma_i^2)^T, \dots, (\gamma_i^N)^T]^T \quad (13)$$

and

$$\Gamma = \{\gamma | \gamma = [\gamma_1, \gamma_2, \dots, \gamma_n]^T \in R^M\}. \quad (14)$$

We now state the FCC problem of the finite-time multi-agent with the switching topology as follows:

Problem 2 Subject to the dynamical system (1) and initial condition (2), find a combined switching vector and consensus parameter vector $(\tau \times \alpha \times \gamma) \in (\Upsilon \times \Lambda \times \Gamma)$ such that the cost function

$$J = t_f + \int_{t_0}^{t_f} \sum_{i=1}^n u_i^2(t) dt \quad (15)$$

is minimized subject to

$$a_{ij}(x_j(t) - x_i(t)) > 0 \quad (16a)$$

$$a_{ij} > 0 \quad (16b)$$

$$0 < \alpha_{ij} < \frac{1}{2} \quad (16c)$$

$$a_{ij} = a_{ji} \quad (16d)$$

$$\alpha_{ij} = \alpha_{ji}, \quad (16e)$$

where

$$u_i = \sum_{j \in N_i} \sum_{k=1}^N a_{ij}^k (x_j - x_i)^{\alpha_{ij}^k} \chi_{[t_{k-1}, t_k)}(t). \quad (17)$$

Remark 1 Constraint (16a) is necessary if control protocol (7) is substituted by (17). However, the constraint (16a) shall be satisfied for all t , which is difficult to deal with, therefore in next section we will develop algorithm to conquer it.

4 Solution Procedure

In Problem 2, the determined variables include weight parameter vector γ and α , and the switching vector τ . This problem is an optimal parameter selection problem of switched system, which is difficult to solve by common optimization method due to the existing of the variable switching times. We first transform the variable switching times into fixed points by TST technique, then solve it by the sequence quadratic programming method.

The transformation from $t \in [t_0, t_f]$ to $s \in [0, N]$ is defined by:

$$\frac{dt(s)}{ds} = v(s) \quad (18)$$

with initial condition

$$t(0) = 0 \quad (19)$$

and the terminal condition

$$t(N) = t_f, \quad (20)$$

where $v(s)$, with possible discontinuity points at $s = 1, \dots, N-1$, is called a time scaling control, which is given by

$$v(s) = \sum_{i=1}^N \delta_i \chi_{[i-1, i)}(s) \quad (21)$$

with $\delta_i \geq 0, i = 1, \dots, N$.

Let $\delta_i, i = 1, \dots, N$, be referred to collectively as δ , and let Δ be the set of all such δ . From (18) and (21), it is clear that

$$t(s) = \int_0^s v(\tau) d\tau = \sum_{j=1}^{i-1} \delta_j + \delta_i (s - i + 1) \quad s \in [i-1, i) \quad (22)$$

and

$$t(N) = \sum_{i=1}^N \delta_i. \quad (23)$$

Define $\tilde{x}_i(s) = x_i(t(s))$. Then, the system (1) and the initial condition (2) are changed into:

$$\dot{\tilde{x}}_i(t) = \tilde{u}_i(s) \times v(s) \quad (24)$$

and

$$\tilde{x}_i(0) = x_i^0. \quad (25)$$

Here,

$$\tilde{u}_i(s) = \sum_{j \in N_i} \sum_{k=1}^N a_{ij}^k (\tilde{x}_j - \tilde{x}_i)^{\alpha_{ij}^k} \chi_{[k-1, k)}(s). \quad (26)$$

The constraint (16a) now is described as:

$$a_{ij}(\tilde{x}_j(s) - \tilde{x}_i(s)) > 0. \quad (27)$$

With the transform (18) and (21), the cost function (15) is converted to:

$$J = \sum_{i=1}^N v_i + \int_0^N \sum_{i=1}^n \tilde{u}_i^2(s) v(s) ds. \quad (28)$$

Now applying the TSC to Problem 2, we obtain the following equivalent problem:

Problem 3 Given the dynamic system (24), with the initial condition (25), find a combined vector $(\delta \times \alpha \times \gamma) \in (\Delta \times \Lambda \times \Gamma)$ such that the cost function (28) is minimized, subject to constraints (27) and (16b)-(16e).

The continuous state inequality constraint (27) is required to be satisfied for all s , which is difficult to solve. Define $\phi(s, \tilde{x}(s)) = a_{ij}(\tilde{x}_i(s) - \tilde{x}_j(s))$. Then constraint (27) can be written as:

$$\phi(s, \tilde{x}(s)) = a_{ij}(\tilde{x}_i(s) - \tilde{x}_j(s)) < 0. \quad (29)$$

By virtue of the inequality transform technique given in [23], the continuous state inequality constraint (29) is approximated by

$$G_\varepsilon(s, \tilde{x}(s)) = \gamma + \int_0^N L_\varepsilon(s, \tilde{x}(s)) ds \leq 0, \quad (30)$$

where

$$L_\varepsilon(s, \tilde{x}(s)) = \begin{cases} \phi(s, \tilde{x}(s)), & \text{if } \phi(s, \tilde{x}(s)) < -\varepsilon \\ -\phi(s, \tilde{x}(s)), & \text{if } -\varepsilon \leq \phi(s, \tilde{x}(s)) \leq \varepsilon \\ 0, & \text{if } \phi(s, \tilde{x}(s)) > \varepsilon. \end{cases}$$

and $\gamma > 0, \varepsilon > 0$.

Problem 3 is now approximated by Problem 4 below.

Problem 4 For $\varepsilon > 0$ and $\gamma > 0$, given dynamic system (24) - (25), find the a combined vector $(\delta \times \alpha \times \gamma) \in (\Delta \times \Lambda \times \Gamma)$ such that the cost function (28) is minimized, subject to constraints (30) and (16b)-(16e).

Problem 4 is a standard optimal parameter selection problem, which can be solved by sequence quadratic programming method. An algorithm is given below:

Algorithm 1

Step 1. We first choose an $\varepsilon > 0$ and a $\gamma > 0$. Then, we solve Problem 4 with such ε and γ . Let $(\sigma^{\varepsilon, \gamma, *}, \delta^{\varepsilon, \gamma, *})$ be the solution obtained.

Step 2. Then, we check if the continuous state inequality constraint (29) is satisfied or not. If it is not satisfied, we will reduce the value of γ to $\frac{\gamma}{2}$ and return to solve Problem 4 with γ taken as $\frac{\gamma}{2}$.

Step 3. We then reduce the value of ε and repeat the process until a satisfactory approximate optimal solution is obtained.

Step 4. By using the transformation (18) and (21), we can obtain the value of vector τ . Then Problem 2 is solved.

Remark 2 [23] shows that, under appropriate assumption, for any $\varepsilon > 0$, there exists a $\gamma(\varepsilon) > 0$ such that if $(\delta \times \alpha \times \gamma) \in (\Delta \times \Lambda \times \Gamma)$ satisfies (30) with γ , $0 < \gamma < \gamma(\varepsilon)$, then it satisfies the continuous state inequality constraint (29). Thus, we see that the reduction step in Algorithm 1 will only require a finite number of times before the continuous state inequality constraint (29) is satisfied.

Remark 3 In this paper, we only consider the case that $a_{ij} = a_{ji} \neq 0$. The general connected topology structure will be discussed in the future work.

5 Simulation

A networked multi-agent system with $n = 4$ members is considered in simulations. Agents are assumed to be equipped with a communication system that allows them to be in contact with their neighbors instantly and exchange their state information without any delay. All simulations are performed in MATLAB. The initial states of the agents are chosen as: $x(0) = [-5, -1, 3, 8]^T$. We consider the case that topology structure changes two times during the whole period. Suppose the switching times are t_1 and t_2 . The control protocol is given as:

$$u_i = \sum_{j \in N_i} \sum_{k=1}^N a_{ij}^k (x_j - x_i)^{\alpha_{ij}^k} \chi_{[t_{k-1}, t_k)}(t), \quad (31)$$

where $k = 1, 2, 3$ and $i = 1, 2, 3, 4$. By using the proposed method, we obtain the following results: The switching times are $t_1 = 1.04$, $t_2 = 3.72$; The convergence time is $t_f = 5.29$; The cost function $J = 26.9931$. The parameter matrix $\{a_{ij}\}$ and $\{\alpha_{ij}\}$ in the three stages are respectively given below:

$$\{\alpha_{ij}^1\} = \begin{bmatrix} 0 & 0.22 & 0.13 & 0.25 \\ 0.22 & 0 & 0.32 & 0.18 \\ 0.13 & 0.32 & 0 & 0.19 \\ 0.25 & 0.18 & 0.19 & 0 \end{bmatrix} \quad (32)$$

$$\{\alpha_{ij}^2\} = \begin{bmatrix} 0 & 0.43 & 0.42 & 0.19 \\ 0.43 & 0 & 0.28 & 0.24 \\ 0.42 & 0.28 & 0 & 0.40 \\ 0.19 & 0.24 & 0.40 & 0 \end{bmatrix} \quad (33)$$

$$\{\alpha_{ij}^3\} = \begin{bmatrix} 0 & 0.49 & 0.53 & 0.34 \\ 0.49 & 0 & 0.68 & 0.22 \\ 0.53 & 0.68 & 0 & 0.57 \\ 0.34 & 0.22 & 0.57 & 0 \end{bmatrix} \quad (34)$$

$$\{\alpha_{ij}^1\} = \begin{bmatrix} 0 & 0.22 & 0.36 & 0.62 \\ 0.22 & 0 & 0.52 & 0.79 \\ 0.36 & 0.52 & 0 & 0.19 \\ 0.62 & 0.79 & 0.19 & 0 \end{bmatrix} \quad (35)$$

$$\{\alpha_{ij}^2\} = \begin{bmatrix} 0 & 0.47 & 0.85 & 0.41 \\ 0.47 & 0 & 0.35 & 0.55 \\ 0.85 & 0.35 & 0 & 0.51 \\ 0.41 & 0.55 & 0.51 & 0 \end{bmatrix} \quad (36)$$

$$\{\alpha_{ij}^3\} = \begin{bmatrix} 0 & 0.27 & 0.35 & 0.19 \\ 0.27 & 0 & 0.75 & 0.12 \\ 0.35 & 0.75 & 0 & 0.30 \\ 0.19 & 0.12 & 0.30 & 0 \end{bmatrix} \quad (37)$$

The state trajectories of agents under protocol (3) are shown in Fig. 1 and the control protocols are shown in Fig. 2. From Fig. 1 we can see that all the agents reach the consensus at 5.29s and the topology structure changes at 1.04s and 3.72s.

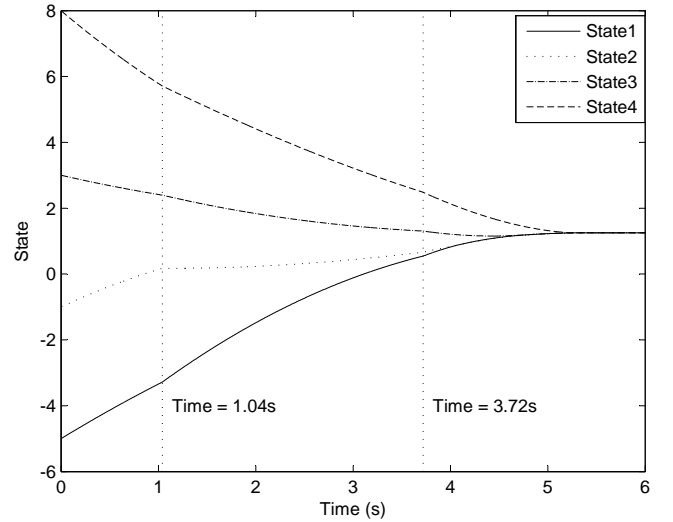


Fig. 1: The state trajectories of agents

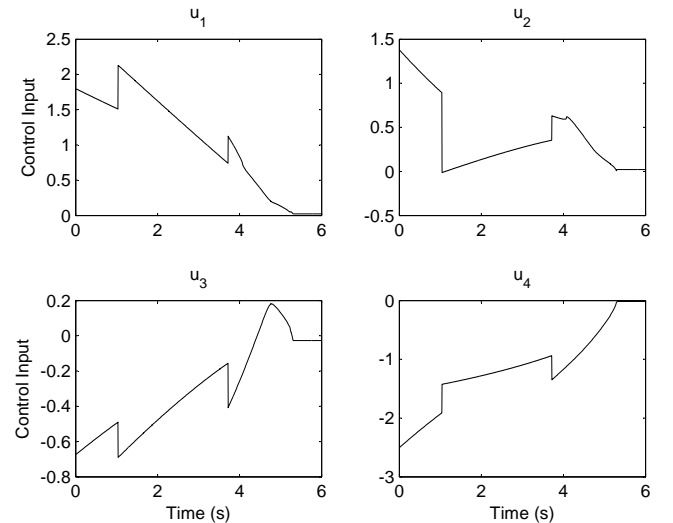


Fig. 2: The control protocols

6 Conclusion

This paper has developed a new finite-time consensus algorithm for multi-agent systems with switching topology. An optimal control problem of switched system was first derived with the continuous state inequality constraints, in which, the weight parameters and the switching points of the topology structure are decision variables. An optimization-based algorithm was obtained by the time scaling transform technique and the constraint transcription method. A series of normal optimal parameter selection problem has been obtained and solved by the sequence quadratic programming method. Simulation results demonstrated the effectiveness of the proposed method.

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