Simulation of Power Grid Networks Considering Wires and Lognormal Leakage Current Variations

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ABSTRACT
As the technology scales into 90nm and below, process-induced variations become more pronounced. In this paper, we propose an efficient stochastic method for analyzing the voltage drop variations of on-chip power grid networks, considering both wire and log-normal leakage current variations. The new analysis is based on the Hermite polynomial chaos (PC) representation of random processes. Different from the existing Hermite PC based method for power grid analysis, which considers only wire variations and model all the random variations as Gaussian processes. The new method consider both wire variations and leakage current variations. We models the variational sub-threshold leakage currents as log-normal distribution random variables. Our experiment results show that the new method is more accurate than the Gaussian-only Hermite PC method using the Taylor expansion method for analyzing leakage current variations, and two orders of magnitude faster than the Monte Carlo method with small variance errors.

1. INTRODUCTION
Process-induced variability has huge impacts on the circuit performance in the sub-90nm VLSI technologies [10, 9]. One important aspect of the variations comes from the chip leakage currents. Leakage currents come from different sources. The dominant factor is the sub-threshold leakage current. The reason is that sub-threshold leakage current has a rapid increasing rate (about 5X-10X increase per technology generation [3]), and it is highly sensitive to threshold voltage V_{th} variations, due to the exponential relationship between sub-threshold current I_{off} and threshold voltage V_{th} as shown below [14],

$$I_{off} = I_{s0} e^{\frac{V_{gs}-V_{th}}{nV_T}} (1 - e^{\frac{-V_{ds}}{V_T}}) \quad (1)$$

where I_{s0} is a constant related to the device characteristics, V_T is the thermal voltage, and n is a constant.

Clearly, the leakage current has exponential dependency on the threshold voltage V_{th}. In the sequel, the leakage current is mainly referred to as the sub-threshold leakage current. Detailed analysis shows that I_{off} is also an exponential function of the channel length L [12]. So, if we model V_{th} or L as the random variables with Gaussian variations due to inter-die or intra-die process variations, then the leakage currents will have a log-normal distribution as shown in [12]. On top of this, those random variables are spatially correlated within a die, due to the nature of the many physical and chemical manufacture processes [9]. But in this paper, we ignore the spacial correlations as they can be dealt with by the orthogonal decomposition [2] or the hierarchical grid modeling approaches [1].

Due to the importance of the impacts on leakage currents on the circuit performances, especially on the on-chip power delivery networks, a number of research works have been proposed recently to perform the stochastic analysis of power grid networks under process-induced leakage current variations. The voltage drop of power grid networks subject to the leakage current variations was first studied in [4, 5]. This method assumes that the log-normal distribution of the node voltage drop is due to log-normal leakage current inputs and is based on a localized Monte Carlo (sampling) method to compute the variance of the node voltage drop. However, this localized sampling method is limited to the static DC solution of power grids modeled as resistor-only networks. Therefore, it can only compute the responses to the standby leakage currents. However, the dynamic leakage currents become more significant, especially when the sleep transistors are intensively used nowadays for reducing leakage powers. In [13, 11], impulse responses are used to compute the means and variances of node voltage responses due to general current variations. But this method needs to know the impulse response from all the current sources to all the nodes, which is expensive to compute for a large network. In [12], the probability density function (pdf) of leakage currents are computed based on the Gaussian variations of channel length.

Recently, a stochastic simulation method for interconnect and power grid networks has been proposed [8, 15]. This method is based on the orthogonal polynomial chaos expansion of random processes to represent and solve for the stochastic responses of linear systems. The major benefit of this method is its compatibility with current transient simulation framework: it solves for some coefficients of the orthogonal polynomials, which can be done by using normal transient simulations of the original circuits with deterministic inputs to compute variances of node responses. Some existing approaches [8] model all the parameter variations as Gaussian (or approximate them as Gaussian variations by using first-order Taylor expansion) [15].

In this paper, we apply the orthogonal polynomial based methods (also called spectral statistical method) to deal with leakage current inputs with log-normal distributions. We show how to represent a log-normal distribution in terms of Hermite polynomials, assuming Gaussian distribution of threshold voltage V_{th} in consideration of intra-die variation. The new method consider both power grid wire and leakage current variations. Experiment results show that the proposed method predicates the variances of the resulting log-normal-like node voltage drops more accurately than Taylor expansion based Gaussian approximation method.

The rest of this paper is organized as follows: Section 2 presents models of power grid networks and the problem we try to solve. Section 3 reviews the orthogonal polynomial chaos based stochastic simulation methods. Section 4 presents our new orthogonal polynomial chaos based method for stochastic analysis power grids subjecting to log-normal leakage current variations. Section 5 presents the framework to consider both wire and leakage current variations. Section 6 presents the experimental results and Section 7 concludes this paper.

2. PROBLEM FORMULATION
In this section, we first present the model of power grids in this paper. We then present the modeling issue of leakage current under intra-die variations. After this, we present the problem that we try to solve.

2.1 Power Grid Network Models

The power grid networks in this paper are modeled as RC networks with known time-variant current sources, which are obtained by gate level logic simulations of the VLSI systems. For a power grid (versus the ground grid), some nodes have known voltage modeled as constant voltage sources. For C4 power grids, the known voltage nodes can be internal nodes inside the power grid. Given known deterministic vector of current sources, $I(t)$, the node voltages can be obtained by solving the following differential equations, which is formulated using modified nodal analysis (MNA) approach,

$$Gv(t) + C\frac{dv(t)}{dt} = I(t) \tag{2}$$

where G is the conductance matrix, C is admittance matrix resulting from capacitive elements. $v(t)$ is the vector of time-varying node voltages and branch currents of voltage sources that we try to solve.

2.2 Modeling Leakage Current Variations

The G and C matrices and input currents $I(t)$ depend on the circuit parameters, such as metal wire width, length, thickness on power grids, and transistor parameters, such as channel length, width, gate oxide thickness, etc. Some previous work assumes that all circuit parameters and current sources are treated as uncorrelated Gaussian random variables [8]. In this paper, we only consider the log-normal leakage current variation, due to the channel length variations, which is modeled as Gaussian (normal) variations [12]. All other circuit parameter variations can be easily considered as shown in [8].

Process-induced variations can also be classified into inter-die (die-to-die) variations and intra-die variations. In inter-die variations, all the parameters variations are correlated. The worst case corner can be easily found by setting the parameters to their top range (mean plus three standard deviations). The difficulty lies in the intra-die variations, where the circuit parameters are not correlated or spatially correlated. Intra-die variations also consist of local and layout dependent deterministic components and random components, which typically are modeled as multivariate Gaussian processes with any spatial correlation [2]. In this paper, we first assume we have a number of transformed ortho-normal random Gaussian variables $\xi(\theta), i = 1,...,n$, which actually model the channel length and the device threshold voltage variations. After that, we consider spatial correlation in the intra-die variation. We apply the principal component analysis method to transfer the correlated variables into un-correlated variables before the spectral statistical analysis.

Let Θ denotes the process sampling space. Let $\theta \in \Theta$, $\xi_i : \theta \rightarrow R$ denotes a normalized Gaussian variable and $\xi(\theta) = [\xi_1(\theta), ..., \xi_n(\theta)]$ is a vector of n independent Gaussian variable. Therefore, given the process variations, the MNA for (2) becomes

$$Gv(t) + C\frac{dv(t)}{dt} = I(t, \xi(\theta)) \tag{3}$$

Note that the input current vector, $I(t,\xi(\theta))$, has both deterministic and random components. In this paper, we assume the dynamic currents (power) due to circuit switching are still modeled as deterministic currents as we only consider the leakage variations.

The problem we need solve is to efficiently find the mean and variances of voltage $v(t)$ at any node and at any time instance. A straightforward method is Monte Carlo (MC) based sampling methods. We randomly generate $I(t,\xi(\theta))$, which is based on the log-normal distribution, solve (3) in time domain for each sampling and compute the means and variances based on sufficient samplings. Obviously, MC will be computationally expensive. However, MC will give the most reliable results and is the most robust and flexible method.

3. SPECTRAL STATISTICAL BASED SIMULATION

In this section, we briefly review the spectral statistical simulation method based on orthogonal polynomial chaos (PC) representation of statistical processes.

3.1 Concept of Hermite Polynomial Chaos

In the following, a random variable $\xi(\theta)$ is expressed as a function of θ, which is the random event. Hermite PC utilizes a series of orthogonal polynomials (with respect to the Gaussian distribution) to facilitate stochastic analysis [16]. These polynomials are used as the orthogonal basis to decompose a random process in a similar way that sine and cosine functions are used to decompose a periodic signal in Fourier series expansion.

For a random variable $v(t,\xi)$ with limited variance, where $\xi = [\xi_1, \xi_2, ...\xi_n]$ is a vector of zero mean ortho-normal Gaussian random variables. The random variable can be approximated by truncated Hermite PC expansion as follows [7]:

$$v(t,\xi) = \sum_{k=0}^{P} a_k H_k^n(\xi) \tag{4}$$

where n is the number of independent random variables, $H_k^n(\xi)$ is n-dimensional Hermite polynomials and a_k are the deterministic coefficients. The number of terms P is given

$$P = \sum_{k=0}^{p} \frac{(n-1+k)!}{k!(n-1)!} \tag{5}$$

where p is the order of the Hermite PC. If only one random variable is considered, the one-dimensional Hermite polynomials are expressed as follows:

$$H_0^1(\xi) = 1, H_1^1(\xi) = \xi, H_2^1(\xi) = \xi^2 - 1, H_3^1(\xi) = \xi^3 - 3\xi, ... \tag{6}$$

Hermite polynomials are orthogonal with respect to Gaussian weighted expectation (the superscript n is dropped for simple notation):

$$< H_i(\xi), H_j(\xi) > = < H_i^2(\xi) > \delta_{ij} \tag{7}$$

where δ_{ij} is the Kronecker delta and $< *, * >$ denotes an inner product defined as follow:

$$< f(\xi), g(\xi) > = \frac{1}{\sqrt{(2\pi)^n}} \int f(\xi) g(\xi) e^{-\frac{1}{2}\xi^T \xi} d\xi \tag{8}$$

Like Fourier series, the coefficient a_k can be found by a projection operation onto the Hermite PC basis:

$$a_k(t) = \frac{< v(t,\xi), H_k(\xi) >}{< H_k^2(\xi) >}, \ \forall k \in \{0, ..., P\}. \tag{9}$$

3.2 Simulation Approach Based on Hermite PCs

In case that $v(t,\xi)$ is unknown random variable vector (with unknown distributions) like node voltages in (3), then the coefficients can be computed by using Galerkin method, which states that the best approximation of $v(t,\xi)$ is obtained when the error $\Delta(t,\xi)$, which is defined as

$$\Delta(t,\xi) = Gv(t) + C\frac{dv(t)}{dt} - I(t, \xi(\theta)) \tag{10}$$

is orthogonal to the approximation. That is

$$< \Delta(t,\xi), H_k(\xi) > = 0, \ i = 0, 1, ..., P \tag{11}$$

In this way, we transform the stochastic analysis process to a deterministic process, where we only need to compute the coefficients of its Hermite PC. Once we obtain those coefficients, the mean and variance of the random variables can be easily computed as shown later in the section.

For illustration purpose, we consider one Gaussian variable $\xi = [\xi_1]$ and assume that the node voltage response can be written as second order ($p = 2$) Hermite PC:

$$v(t,\xi) = v_0(t) + v_1(t)\xi_1 + v_2(t)(\xi_1^2 - 1) \quad (12)$$

assuming that the input leakage current sources can also be represented by a second Hermite PC:

$$I(t,\xi) = I_0(t) + I_1(t)\xi_1 + I_2(t)(\xi_1^2 - 1) \quad (13)$$

By applying the Galerkin equation (11) and the orthogonal property of the various order of Hermite PCs, we end up with the following equations

$$Gv_i(t) + C\frac{dv_i(t)}{dt} = I_i(t) \quad (14)$$

where $i = 0,1,2,..,P$. For two independent Gaussian variables, we have

$$\begin{aligned} v(t,\xi) &= v_0(t) + v_1(t)\xi_1 + v_2(t)\xi_2 + v_3(t)(\xi_1^2 - 1) \\ &\quad + v_4(t)(\xi_2^2 - 1) + v_5(\xi_1\xi_2) \end{aligned} \quad (15)$$

Assuming that we have a similar second order Hermite PC for input leakage current $I(t,\xi)$,

$$\begin{aligned} I(t,\xi) &= I_0(t) + I_1(t)\xi_1 + I_2(t)\xi_2 + I_3(t)(\xi_1^2 - 1) \\ &\quad + I_4(t)(\xi_2^2 - 1) + I_5(\xi_1\xi_2) \end{aligned} \quad (16)$$

The (14) is valid with $i = 0,\ldots,5$. For more (more than two) Gaussian variables, we can obtained the similar results with more coefficients of Hermite PCs to be solved by using (14).

Once we obtain the Hermite PC of $v(t,\xi)$, we can obtain the mean and variance of $v(t,\xi)$ trivially as (one Gaussian variable case):

$$\begin{aligned} E(v(t,\xi)) &= v_0(t) \\ Var(v(t,\xi)) &= v_1^2(t)Var(\xi_1) + v_2^2(t)Var(\xi_1^2 - 1) \\ &= v_1^2(t) + 2v_2^2(t) \end{aligned} \quad (17)$$

One critical problem remains so far is how to obtain the Hermite PC (13) for leakage current with log-normal distribution. This will be explained in details in the next section.

4. LOG-NORMAL LEAKAGE CURRENT VARIATIONS

In this section, we present the new method for representing the log-normal leakage current distributions by using Hermite PCs with one or more independent Gaussian variables representing the channel length or threshold voltage variations.

Our method is based on [6] and we will show how it can be applied to solve our problems for one or more independent Gaussian variables.

4.1 Hermite Polynomial Chaos representation

Let $g(\xi)$ be the Gaussian random variable, denoting threshold voltage or device channel length. Let $l(\xi)$ be the random variable obtained by taking the exponential of $g(\xi)$

$$l(\xi) = e^{g(\xi)},\ g(\xi) = ln(l(\xi)) \quad (18)$$

Obviously, for the MOS device leakage current equation (1), leakage current, $I_{off} = cI_l(V_{th}) = ce^{-V_{th}}$, where the leakage component $I_l(V_{th})$ is a log-normal random variable. Let the mean and the variance of $g(\xi)$ as μ_g and σ_g^2, then the mean and variance of $l(\xi)$ are as follows:

$$\mu_l = e^{(\mu_g + \frac{\sigma_g^2}{2})} \quad (19)$$

$$\sigma_l^2 = e^{(2\mu_g + \sigma_g^2)}[e^{\sigma_g^2} - 1] \quad (20)$$

For general Gaussian variable $g(\mathbf{x})$, it can be represented as

$$g(\mathbf{x}) = \sum_{i=0}^{n} \xi_i g_i \quad (21)$$

where ξ_i are orthonormal Gaussian variables. i.e. $< \xi_i, \xi_j >= \delta_{ij}$, $< \xi_i >= 0$ and $\xi_0 = 1$. Note that such form can always be obtained by using Karhunen-Loeve orthogonal expansion method [7]

In our problem, we need to represent the log-normal random variable $l(\xi)$ by using the Hermite PC expansion form:

$$l(\xi) = \sum_{k=0}^{P} l_k H_k^n(\xi) \quad (22)$$

where $l_0 = \exp[\mu_g + \frac{\sigma_g^2}{2}]$. To find the other coefficients, we can apply (9) on $l(\xi)$. Therefore, we have

$$l_k(t) = \frac{< l(t,\xi), H_k(\xi) >}{< H_k^2(\xi) >},\ \forall k \in \{0,\ldots,P\}. \quad (23)$$

It was shown in [6], $l(\xi)$ can be written as

$$l(\xi) = \frac{< H_k(\xi - \mathbf{g}) >}{< H_i^2(\xi) >} = \exp[\mu_g + \frac{1}{2}\sum_{j=1}^{n} g_j^2] \quad (24)$$

where n is the number of independent Gaussian random variables.

The log-normal process can then be written as

$$l(\xi) = l_0(1 + \sum_{i=1}^{n}\xi_i g_i + \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(\xi_i\xi_j - \delta_{ij})}{< (\xi_i\xi_j - \delta_{ij})^2 >} g_i g_j + \ldots) \quad (25)$$

where g_i is defined in (21).

4.2 Hermite PC with one Gaussian variable

In this case, $\xi = [\xi_1]$. For the second order Hermite PC ($P = 2$), following (25), we have

$$l(\xi) = l_0(1 + \sigma_g\xi_1 + \frac{1}{2}\sigma_g^2(\xi_1^2 - 1)) \quad (26)$$

Hence, the desired Hermite PC coefficients, $I_{0,1,2}$, can be expressed as $l_0, l_0\sigma_g$ and $\frac{1}{2}l_0\sigma_g^2$ respectively.

4.3 Hermite PC with two and more Gaussian variables

For two random variables ($n = 2$), assume that $\xi = [\xi_1, \xi_2]$ is a normalized uncorrelated Gaussian random variable vector that represents random variable $g(\xi)$:

$$g(\xi) = \mu_g + \sigma_1\xi_1 + \sigma_2\xi_2 \quad (27)$$

Note that

$$< (\xi_i\xi_j - \delta_{ij})^2 >=< \xi_i^2\xi_j^2 >=< \xi_i^2 >< \xi_j^2 >= 1$$

Therefore, the expansion of the log-normal random variables using second order Hermite PCs can be expressed as

$$\begin{aligned} l(\xi) &= l_0(1 + \sigma_1\xi_1 + \sigma_2\xi_2 + \frac{\sigma_1^2}{2}(\xi_1^2 - 1) + \frac{\sigma_2^2}{2}(\xi_2^2 - 1) + \\ &\quad 2\sigma_1\sigma_2\xi_1\xi_2) \end{aligned} \quad (28)$$

where

$$\mu_l = l_0 = \exp(\mu_g + \frac{1}{2}\sigma_1^2 + \frac{1}{2}\sigma_2^2)$$

Hence, the desired Hermite PC coefficients, $I_{0,1,2,3,4,5}$, can be expressed as $l_0, l_0\sigma_1, l_0\sigma_2, \frac{1}{2}l_0\sigma_1^2, \frac{1}{2}l_0\sigma_2^2$, and $2l_0\sigma_1\sigma_2$ respectively.

Similarly, for four Gaussian random variables, assume that $\xi = [\xi_1, \xi_2, \xi_3, \xi_4]$ is a normalized, uncorrelated Gaussian random variable vector. The random variable $g(\xi)$ can be expressed as

$$g = \mu_g + \sum_{i=1}^{4} \sigma_i \xi_i \tag{29}$$

As a result, the log-normal random variable $l(\xi)$ can be expressed as

$$l(\xi) = l_0(1 + \sum_{i=1}^{4} \xi_i \sigma_i + \sum_{i=1}^{4} \frac{1}{2}(\xi_i^2 - 1)\sigma_i^2 + \sum_{i=1}^{4}\sum_{j=1}^{4} \xi_i \xi_j \sigma_i \sigma_j + ...) \tag{30}$$

where

$$\mu_l = l_0 = \exp(\sigma_0 + \frac{1}{2}\sum_{i=1}^{4} \sigma_i^2)$$

Hence, the desired Hermite PC coefficients can be expressed using the equation (30) above.

Once we have the Hermite PC representation of the leakage current sources $I(t, \xi)$, the node voltages $v(t, \xi)$ can be computed by using equations (14) with proper order p of the PCs to obtain all the Hermite PC coefficients of $v(t, \xi)$.

5. VARIATIONS IN WIRES AND LEAKAGE CURRENTS

In this section, we will consider variations in width (W), thickness(T) of wires of power grids, as well as threshold voltage(V_{th}) in active devices which are reflected in the leakage currents. Meanwhile, without loss of generality, these variations are supposed to be independent of each other. As mentioned in [8], the MNA equation for the ground circuit turns to be:

$$G(\xi_g)v(t) + C(\xi_c)\frac{dv(t)}{dt} = I(\xi_I, t) \tag{31}$$

The variation in width W and thickness T will cause variation in conductance matrix G and capactance matrix C while variation in threshold voltage will cause variation in leakage currents. Thus, the conductances and capactances of wires can be expressed as in [8]:

$$\begin{aligned} G(\xi_g) &= G_0 + G_1\xi_g \\ C(\xi_c) &= C_0 + C_1\xi_c \end{aligned} \tag{32}$$

G_0, C_0 represents the deterministic components of conductance and capactances of the wires. G_1, C_1 represents sensitivity matrices of the conductances and capactances . ξ_g, ξ_c are normalized random variables with guassian distribution. As mentioned in previous section, the variation in leakage current resulting can be represented by a second Hermite PC as in equation (26):

$$I(t, \xi_I) = I_0(t) + I_1(t)\xi_I + I_2(t)(\xi_I^2 - 1) \tag{33}$$

here, ξ_i is a normalized Guassian distribution random variable representing variation in threshold voltage. Hence, $I(t, \xi_I)$ follows log-normal distribution as

$$\begin{aligned} I &= e^{g(\xi_I)} \\ g(\xi_I) &= \mu_I + \sigma_I \xi_I \end{aligned} \tag{34}$$

As in previous part, the desired Hermite PC coefficients, $I_{0,1,2}$, can be expressed as $l_0, l_0\sigma_I$ and $\frac{1}{2}l_0\sigma_I^2$ respectively. I_0 is the mean of leakage current source, which is expressed as

$$I_0 = exp(\mu_I + \frac{1}{2}\sigma_I^2) \tag{35}$$

Therefore, with the influence of ξ_g, ξ_c, ξ_I, the node voltage is expended by Hermite PC in the second order as

$$\begin{aligned} v(t,\xi) \;=\; & v_0(t) + v_1(t)\xi_g + v_2(t)\xi_c + v_3(t)\xi_I \\ & + v_4(t)(\xi_g^2 - 1) + v_5(t)(\xi_c^2 - 1) + v_6(t)(\xi_I^2 - 1) \\ & + v_7(t)\xi_g\xi_c + v_8(t)\xi_g\xi_I + v_9(t)\xi_c\xi_I \end{aligned} \tag{36}$$

Now the task is to get coeffients of the HPC of node voltage $v(t, \xi)$. Applying the Galerkin equation (11), we will get the equations as

$$\begin{array}{ccc} & <\Delta(t,\xi), 1>= 0; & \\ <\Delta(t,\xi), \xi_g>= 0; & <\Delta(t,\xi), \xi_c>= 0; & <\Delta(t,\xi), \xi_I>= 0; \\ <\Delta(t,\xi), \xi_g^2 - 1>= 0; & <\Delta(t,\xi), \xi_c^2 - 1>= 0; & <\Delta(t,\xi), \xi_I^2 - 1>= 0; \\ <\Delta(t,\xi), \xi_g\xi_c>= 0; & <\Delta(t,\xi), \xi_g\xi_I>= 0; & <\Delta(t,\xi), \xi_c\xi_I>= 0; \end{array} \tag{37}$$

With the distribution of $\mathbf{\xi_g, \xi_c, \xi_I}$, we can get these coeffients $v(t) = [v_0(t), v_1(t), ..., v_9(t)]^T$ of node voltage as

$$\widetilde{G}v(t) + \widetilde{C}\frac{dv(t)}{dt} = \widetilde{I}(t) \tag{38}$$

where

$$\widetilde{G} = \begin{bmatrix} G_0 & G_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ G_1 & G_0 & 0 & 0 & 2G_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & G_0 & 0 & 0 & 0 & 0 & G_1 & 0 & 0 \\ 0 & 0 & 0 & G_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & G_1 & 0 & 0 & G_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & G_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & G_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & G_0 & 0 & 0 \\ 0 & 0 & 0 & G_1 & 0 & 0 & 0 & 0 & G_0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & G_0 \end{bmatrix}$$

$$\widetilde{C} = \begin{bmatrix} C_0 & 0 & C_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & C_0 & 0 & 0 & 0 & 0 & 0 & C_1 & 0 & 0 \\ C_1 & 0 & C_0 & 0 & 0 & 2C_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_1 & 0 & 0 & C_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & C_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & C_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & C_0 & 0 \\ 0 & 0 & 0 & C_1 & 0 & 0 & 0 & 0 & 0 & C_0 \end{bmatrix}$$

$$\widetilde{I}(t) = [I_0(t), 0, 0, I_1(t), 0, 0, I_2(t), 0, 0, 0]^T$$

With the HPC coeffients of node voltage $v(t, \xi)$, it is easy to get the mean and variance of $v(t, \xi)$, which describe the random characteristic of node voltages in the given circuit.

6. EXPERIMENTAL RESULTS

This section describes the simulation results of circuits with log-normal leakage current distributions for a number of power grid networks. All the proposed methods have been implemented in Matlab. All the experimental results are carried out in a Linux system with dual Intel Xeon CPUs with 3.06Ghz and 1GB memory.

6.1 Comparison with Taylor expansion method

We first compare the proposed method with the simple Taylor expansion method for one and more Gaussian variables.

For simplicity, we assume one Gaussian random variable $g(\xi)$, which is expressed as

$$g = \mu_g + \sigma_g \xi \tag{39}$$

where ξ is a normalized Gaussian random variable with $<\xi> = 0$, and $<\xi^2> = 1$. The log-normal random variable $l(\xi)$, obtained

Table 1: Accuracy comparison between Hermite PC (HPC) and Taylor Expansion

δ_g	0.01	0.1	0.3	0.5	0.7
HPC (%)	3.19	1.88	2.07	5.5	2.92
Taylor (%)	3.19	1.37	2.41	16.6	24.02

from $g(\xi)$, is written as

$$l(\xi) = e^{g(\xi)} = \exp(\mu_g + \sigma_g \xi) \tag{40}$$

Expand the exponential into Taylor series and keep all the terms up to second order, then we have

$$\begin{aligned} l(\xi) &= 1 + \sum_{i=0}^{1} \xi_i g_i + \frac{1}{2}\sum_{i=0}^{1}\sum_{j=0}^{1} \xi_i \xi_j g_i g_j + \ldots \\ &= 1 + \mu_g + \frac{1}{2}\mu_g^2 + \frac{1}{2}\sigma_g^2 + (\sigma_g + \mu_g \sigma_g)\xi + \\ &\quad \frac{1}{2}\sigma_g^2(\xi^2 - 1) + \ldots \end{aligned} \tag{41}$$

We observe that the second-order Taylor expansion, as shown in (40), is similar to second order Hermite PC in (28). Hence, the Galerkin method can still be applied, we then use (14) to obtain the Hermite PC coefficients of node voltage $v(t, \xi)$ accordingly. We want to emphasize, however, that the polynomials generated by Taylor expansion in general are not orthogonal with respect to Gaussian distributions and can't be used with Galerkin method, unless we only keep the first order of Taylor expansion results (with less accuracy). In this case, the resulting node voltage distribution is still Gaussian, which obviously is not correct.

We note that the first order Taylor expansion has been used in the statistical timing analysis [2]. The delay variations, due to interconnects and devices, can be approximated with this limitation. The skew distributions may be computed easily with Gaussian process.

To compare these two methods, we use the Monte Carlo method as to measure the accuracies of two methods in terms of standard deviation. For Monte Carlo, we sample 2000 times and the results are summarized in Table 1. In this table, δ_g is the standard deviation of the Gaussian random threshold voltage Gaussian variable in the log-normal current source. *HPC* is the standard deviation from the Hermite PC method in terms of relative percentage against the MC method. *Taylor* is the standard deviation from the Taylor expansion method in terms of relative percentage against the MC method. We can observe that when the variation of current source increases, the Taylor expansion method will result in significant errors compared to the MC method, while the proposed method has the smaller errors for all cases. This clearly shows the advantage of the proposed method.

6.2 Experiment considering leakage current variation

Fig.1 shows the node voltage distributions at one node of a ground network with 1720 nodes. The Monte Carlo results are obtained by 2000 samples. The standard deviations of the log-normal current sources with one Gaussian variable is 0.1. The mean and 3δ (standard deviations) computed by the Hermite PC method are also marked in the figure which fits very well with the MC results.

To consider multiple random variables, we divide the circuit into several partitions. We first divide the circuit into two parts. Fig. 2 shows the node voltage of one node of a particular time instance of a ground network with 336 nodes with two independent variables. The standard deviations for two Gaussian variations are $\sigma_{g1} = 0.5$, $\sigma_{g2} = 0.1$. The 3δ variations are also marked in the figure. Table 2 shows the speedup of the Hermite PC method over Monte Carlo method with 3000 samples. In this table, *#node* is the number of

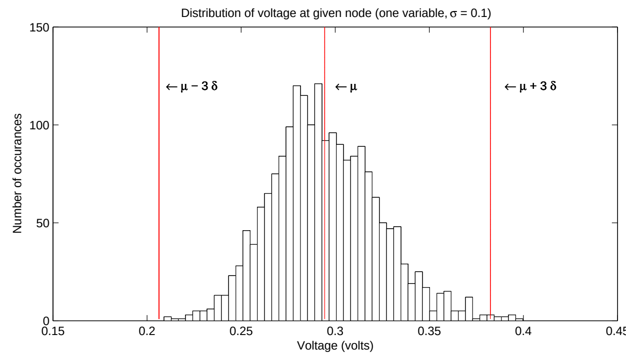

Figure 1: Distribution of the voltage in a given node with one Gaussian variable, $\sigma_g = 0.1$

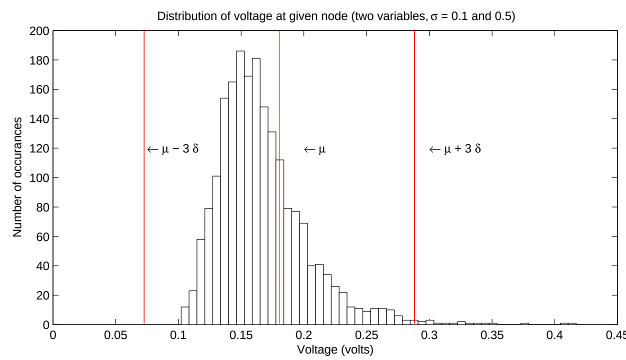

Figure 2: Distribution of the voltage in a given node with two Gaussian variables, $\sigma_{g1} = 0.1$ **and** $\sigma_{g2} = 0.5$

nodes in the power grid circuits. p is the order of the Hermite PCs and n is the number of independent Gaussian random variables. HPC and MC represent the CPU times in seconds used for Hermite PC and MC method respectively. It can be seen that the proposed method is about two order of magnitude faster than the MC method. When more Gaussian variables are used for modeling intra-die variations, we need more Hermite PC coefficients to compute. Hence, the speedup will be smaller if the MC method uses the same number of samples as shown in *gridrc_12*. Also, one observation is that the speedup depends on the sampling size in MC method. We found that 2000 to 3000 samples are the reasonable numbers to have good MC results. Note that the large-sized circuit, such as *gridrc_67*, is unable to finish within reasonable time using MC. The advantage of HPC is obvious as shown in the table.

6.3 Experiment considering variation in G,C,I

Considering variation in conductance, capacitor and leakage current at the same time, Fig. 3 and fig. 4 show the node voltage distributions at one node of two different ground circuit, Circuit1 and Circuit2, respectively. Circuit1 contains 52 nodes and Circuit2 contains 280 nodes. In the figures, the blue line is the mean voltage and worst case voltage using HPC method. The write hist is the Monte Carlo results of 2000 samples. The red line is the mean voltage

Table 2: CPU time comparison with the Monte Carlo method

Ckt	#node	p	n	MC(s)	HPC(s)	Speedup
gridrc_3	33	2	1	5.39	0.006	898.33
gridrc_10	1720	2	1	60506.25	61.48	984.16
gridrc_12	3024	2	2	3.13×10^5	625.63	499.61
gridrc_67	7400	2	2	N/A	1979	N/A

Table 3: CPU time comparison with the Monte Carlo method considering variation in G,C,I

Ckt	#node	MC(s)	HPC(s)	Speedup
gridrc_6	52	365.57	1.02	358.4
gridrc_61	280	9229	68.59	134.6
gridrc_62	645	39587	741.02	53.4

and worst case voltage of the 2000 sameples. We can see that they match very well. Table 3 shows the CPU speedup of *HPC* method

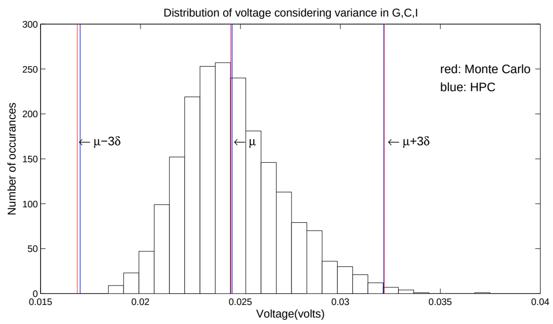

Figure 3: Distribution of the voltage in a given node in circuit1 with variation on G,C,I

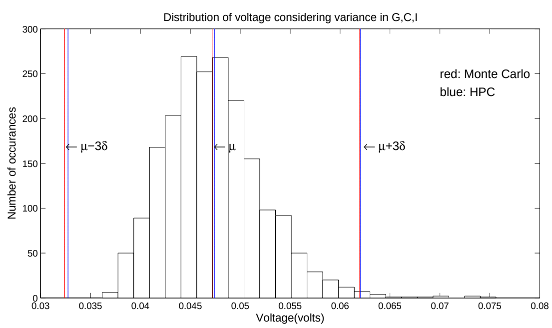

Figure 4: Distribution of the voltage in a given node in circuit2 with variation on G,C,I

than *MC* method. The sample time of Monte Carlo is 3500 and we can see that it's more than 100X time faster of proposed method than the Monte Carlo method. The advantage is obviously seen in the table.

7. CONCLUSION AND FUTURE WORK

In this paper, we have proposed a new stochastic simulation method for fast estimating the voltage variations due to the process-induced log-normal leakage current variations. The new analysis is based on the Hermite polynomial chaos (PC) representation of random processes. We extended the existing Hermite PC based power grid analysis method [8] by considering log-normal leakage distribution as well as wire variations due to process variations. Our experimental results show that the new method is more accurate than the Gaussian-only Hermite PC using the Taylor expansion method for analyzing leakage current variations and two orders of magnitude faster than Monte Carlo methods with small variation errors. The proposed method leads to about 1% or less of errors in both mean and standard deviations.

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