

Battery State of Charge Estimation using Adaptive Subspace Identification Method

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Abstract

Estimation of battery state of charge (SOC) is essential for many emerging battery powered applications such as smart phones, electric and hybrid electric vehicles. In this paper, we propose a new battery SOC estimation method using adaptive subspace identification method. The subspace identification method is a numerically robust approach and is used to build the dynamic linear model based on battery's terminal voltages and current. To deal with the non-linearity of the battery, the transient battery terminal voltages and current are partitioned into piecewise linear regions and subspace identification is performed on each linear region. As a result, the battery SOC can be accurately calculated for each region. Our experiments show that the new method has an error margin of 1.4% from ideal SOC values as given by *Dualfoil*, a powerful battery simulator. This outperforms the least square estimation algorithm, which is found to have a higher error margin of 4.5% for some load profiles, while not converging at all for some other load profiles.

1. Introduction

The state of charge (SOC) of the battery is a measure of the amount of electrical energy stored in it. Accurate SOC estimation is critical for proper operation of the battery. In the case of Electric Vehicles (EV) and Hybrid Electric Vehicles (HEV), incorrect SOC estimation can lead to over-charging or under-charging of the battery resulting in battery failure.

A linear relationship exists between the SOC of the battery and its open circuit voltage (V_{oc}) [1].

$$S(t) = \frac{V_{oc}(t) - V_{bs0}}{V_{bs100}} \quad (1)$$

where $S(t)$ is the SOC of the battery expressed as a percentage, V_{bs0} is the battery terminal voltage when the battery is completely discharged (SOC = 0%) and V_{bs100} is the battery terminal voltage when the battery is completely charged (SOC = 100%). From (1), it is seen that the estimation of SOC is equivalent to the estimation of the V_{oc} .

Many approaches have been proposed in the past to estimate the battery's SOC. The SOC estimation for lead acid batteries is discussed for stationary and non-stationary applications in [5] and [6] respectively. In [1], the authors developed an extended Kalman filter to estimate the SOC of the battery using open circuit battery voltage method. In [2], the authors used the equivalent circuit model developed in [1] and designed an adaptive least square estimator to determine the V_{oc} of the battery, but no experimental results were provided. More recently, authors in [7] did not use any circuit model but instead used a linear system analysis to estimate the V_{oc} and hence, the SOC.

In this paper, a new SOC estimation method is presented which determines the V_{oc} (and therefore, the SOC) using an adaptive subspace identification method. The equivalent circuit model developed in [1] is formulated as a piecewise linear time invariant system. The subspace for this system is determined using the subspace identification algorithm [4]. Once the subspace has been identified, V_{oc} is estimated for different current waveforms. The results obtained are compared with the ideal values as well as the least square estimator described in [2].

This paper is organized as follows: Section 2 describes the battery circuit model. Section 3 describes the subspace identification method and how it can be used for determining the battery open circuit voltage. Experimental results based on this method are presented in Section 4. Conclusion is presented in Section 5.

2. Battery Circuit Model

The reactive battery circuit model for an electrochemical battery is shown in Figure 1 [1].

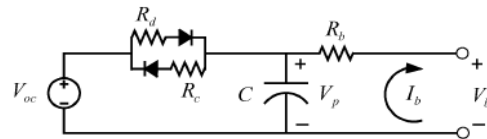


Figure 1: Circuit Model used for the Battery

The dynamic model expression for the battery circuit can be written as:

$$\begin{aligned}\dot{V}_p &= -\frac{V_p}{R_d C} + \frac{V_{oc}}{R_d C} - \frac{I_b}{C}, V_p \leq V_{oc} \\ \dot{V}_p &= -\frac{V_p}{R_c C} + \frac{V_{oc}}{R_c C} - \frac{I_b}{C}, V_p \geq V_{oc} \\ V_b &= V_p - I_b R_b\end{aligned}\quad (2)$$

$$(3)$$

where V_{oc} is the ideal voltage representing the open circuit voltage, V_b is the battery terminal voltage, V_p is the voltage across the capacitor, C is the polarization capacitance, I_b is the current at the battery terminals and R_b is the terminal resistance. R_d is the equivalent internal resistance when the battery is discharging ($I_b > 0$) and R_c is the equivalent internal resistance when the battery is charging ($I_b < 0$). The battery circuit parameters R_b , R_c , R_d and C are not known and the voltage V_p which is internal to the battery is not directly measurable.

Previous research has modeled the battery circuit as a linear time varying system in order to estimate the V_{oc} [1, 2]. In this paper, the battery is modeled as a piecewise linear dynamic system and the V_{oc} is estimated.

3. Subspace Identification Method

In this section, the subspace identification method [4], which is used to model the battery's SOC for each linear region as a linear dynamic subsystem, is briefly presented. The state space model for a linear system can be written in discrete form as:

$$\begin{aligned}x(t+1) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t)\end{aligned}\quad (4)$$

where $A \in R^{n \times n}$, $B \in R^{n \times m}$, $C \in R^{1 \times n}$ and $D \in R^{1 \times m}$. The input vector is $u(t) \in R^{m \times 1}$ and the output vector is $y(t) \in R^{1 \times 1}$.

3.1. The N4SID Algorithm

The N4SID algorithm [4] is used to determine the SOC of the battery by determining the open circuit voltage, V_{oc} . From (4), the matrix input-output equations are derived as:

$$Y_p = O_i X_p + T_i U_p \quad (5)$$

$$Y_f = O_i X_f + T_i U_f \quad (6)$$

Where X_p is the past state sequence, X_f is the future state sequence and T_i is the block Toeplitz matrix. A property important for subspace identification:

$$Pu_f(Y_f, W_p) = O_i X_f \quad (7)$$

Can be decomposed as:

$$Y_f = Pu_f(Y_f, W_p) + Pw_p(Y_f, U_f) \quad (8)$$

From (6), it can be seen that Y_f can be decomposed as the direct sum of $O_i X_f$ and $T_i U_f$. As a result, $O_i X_f$ lies in the row space of W_p and $T_i U_f$ lies in the row space of U_f . Applying Singular Value Decomposition (SVD) on the left hand side of (7),

$$Pu_f(Y_f, W_p) = [U_1 \quad U_2] \begin{bmatrix} \Sigma_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} = U_1 \Sigma_1 V_1^T \quad (9)$$

The extended observability matrix O_i and the future state sequence X_f can be identified as:

$$O_i = U_1 \Sigma_1^{1/2} T \quad (10)$$

$$X_f = T^{-1} \Sigma_1^{1/2} V_1^T \quad (11)$$

where T is an arbitrary non-singular matrix. Once the state sequence $X_f = X_i = [x(i), x(i+1), \dots, x(i+N-1)]$ is identified, the A , B , C and D matrices can therefore be determined.

3.2. Battery State of Charge Estimation using Subspace Identification Method

The discharging case of the battery is considered in this paper. The battery equations for the discharging case can be written as:

$$\dot{V}_p = -\frac{V_p}{R_d C} + \frac{V_{oc}}{R_d C} - \frac{I_b}{C}, V_p \leq V_{oc} \quad (12)$$

$$V_b = V_p - I_b R_b \quad (13)$$

The state space variables can be defined as:

$$x_1 = V_p, \quad x_2 = \frac{V_{oc}}{R_d C} \quad (14)$$

The battery circuit model can then be written as:

$$\dot{x}_1 = -\frac{x_1}{R_d C} + x_2 - \frac{u(t)}{C} \quad (15)$$

$$\dot{x}_2 = 0 \quad (16)$$

$$y_1 = x_1 - R_b u(t) \quad (17)$$

$$y_2 = R_d C x_2 \quad (18)$$

Equations (15) – (18) define a linear time varying system, where $u(t) = I_b(t)$ is the input and $y_1(t) = V_b(t)$ and $y_2(t) = V_{oc}(t)$ are the outputs. (16) is obtained assuming V_{oc} is a slow time varying signal such that $\dot{V}_{oc} \approx 0$ [2].

A practical electro-chemical battery is however, rarely linear as its properties change over time. This is confirmed by the *Dualfoil* battery simulator [3].

A non-linear time varying model can be approximated as a piecewise linear dynamic system. The only restriction is that the piecewise functions should not have any discontinuities [4]. Therefore, for every segment of the piecewise linear system, the subspace identification method can be used to determine the A , B , C and D matrices. The subspace can then be trained for m piecewise segments until the subspace identification converges. The trained system can be used to accurately determine the open circuit voltage V_{oc} by just knowing the input current waveform I_b .

4. Experimental Results

The proposed method is implemented in Matlab 7.0. As a comparison, the least square estimation method [2] was implemented and the results obtained were compared with those obtained from N4SID method [4]. The experimental results were obtained on Linux server with quad-core Intel Xeon CPU and 16GB memory.

The simulations were carried out for a lithium ion battery. For one particular current I_b , the voltages V_b and V_{oc} that are obtained using *Dualfoil* battery simulator are fed piecewise into the subspace identification system in order to train the system. The trained subspace can then be used to estimate the V_{oc} for different current profiles for the same battery type. We used four different input current profiles for verification of the proposed method: a) Periodic Discharge, b) Piecewise Continuous, c) Constant Load and d) Constant Power.

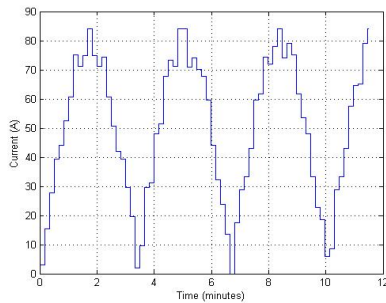


Figure 2: Periodic Discharge input current profile

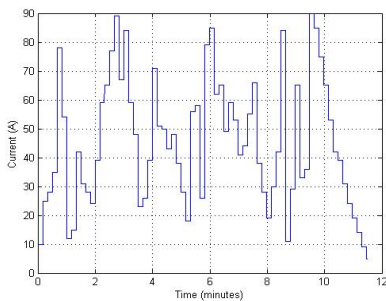


Figure 3: Piecewise Continuous input current profile

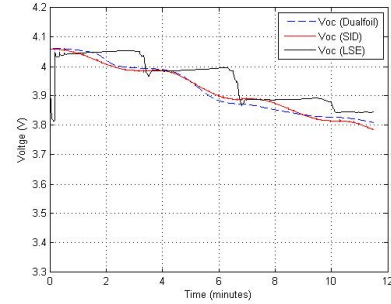


Figure 4: V_{oc} for Periodic Discharge current profile

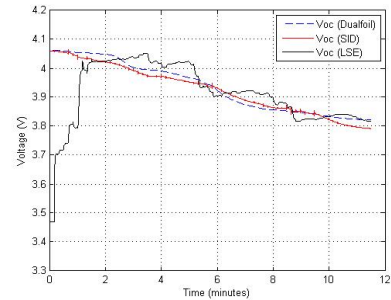


Figure 5: V_{oc} for Piecewise Continuous current profile

Figures 2 and 3 show the periodic discharge and piecewise continuous input current profiles respectively. Figures 4 and 5 show the V_{oc} obtained using the subspace identification method and how it compares to the actual values from *Dualfoil* and the least square estimator for periodic discharge and piecewise continuous input current profiles respectively. For both cases, it is seen that subspace identification estimates the V_{oc} very accurately. Figures 4 and 5 show that the least square estimator takes some time to converge initially for periodic discharge as well as for piecewise continuous current profiles, and when it does, it still shows an error higher than that of the subspace identification method.

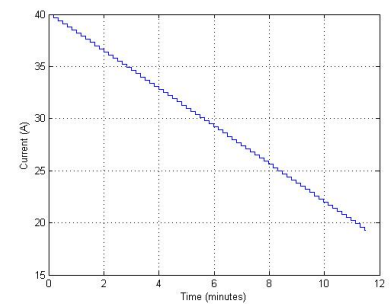


Figure 6: Constant Load input current profile

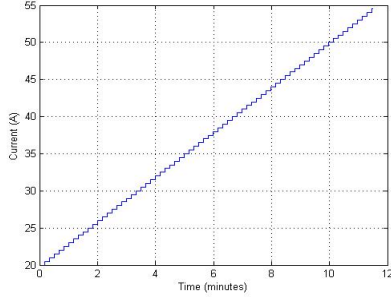


Figure 7: Constant Power input current profile

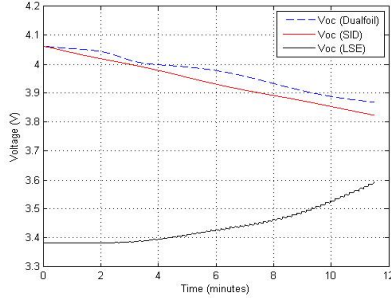


Figure 8: V_{oc} for Constant Load current profile

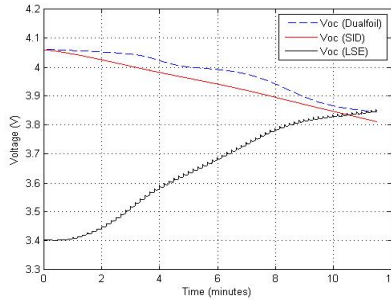


Figure 9: V_{oc} for Constant Power current profile

Figures 6 and 7 show the constant load and constant power input current profiles respectively. Figures 8 and 9 compare the V_{oc} values obtained using the subspace identification method and the least square estimation method for constant load and constant power input current profiles. For both these inputs, it can be seen that least square estimator doesn't converge at all. In contrast, the subspace identification method estimates the V_{oc} values accurately for these profiles as well.

Table 1: Maximum Error for different current profiles

Input Current Profile	Maximum Error using SID	Maximum Error using LSE
Periodic discharge	0.7%	4.5%
Piecewise constant	0.8%	2.5%
Constant load	1.2%	19.1%
Constant power	1.4%	19.1%

Table 1 shows the maximum error obtained for all the four input current profiles for both subspace identification method as well as least square estimation method. It is seen that for the proposed subspace identification method, the maximum error observed across all the different load profiles is 1.4%.

5. Conclusion

In this paper, an adaptive subspace identification method was proposed to identify the open circuit voltage V_{oc} (and hence, the SOC) for the discharging scenario of a battery. The robustness of the proposed approach was verified using four different load profiles and compared with actual values from *Dualfoil* as well to the least square estimation method [2]. For each one, experimental results show that the subspace identification method estimates V_{oc} accurately with maximum error of 1.4%. In contrast, the least square estimation method [2] shows a higher error margin of 4.5% for some load profiles, while not converging at all for others.

Future work will investigate the efficacy of this approach on various other battery types.

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